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Shaping the odds: insecurity management in general equilibrium

by
M.A. Keyzer

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Contents

Abstract

Insecurity management differs from risk management in that it takes the distribution of uncertain events to be under the control of economic agents, rather than given. This effectively turns these distributions into public goods for all who are affected by them, and therefore calls for dedicated institutional arrangements, to avoid market failure and inefficiency. Once distributions become variable, market failure may also result via the product of the density and the utility function in expected utility maximization, that loses concavity. The paper discusses two ways to maintain concavity, one in which the actions other than those that shape the distributions can be postponed until after uncertainty has been revealed, another in which new institutions restrict the possible probability profiles to a finite number of alternatives. Under central planning, this can be effectuated via standardization. Standard-based labeling offers a decentralized solution that enables individuals to compose a mix from given profiles, either collectively, or individually.

Section 1
Introduction

The management of insecurity is concerned with the provision of opportunities and the prevention of hazards. It plays a role in virtually every sphere of life, including prevention of illness, ensuring food safety, fighting crime, investing in employability, or promoting fundamental research. It differs from the more usual management of risk in that it takes the distribution of uncertain events to be under the control of economic agents, rather than given.

The distinction is not new. In reaction to Hirschleifer (1970), who argues that it is always possible to express uncertainty by means of fixed probabilities, Ehrlich and Becker (1972) point out that it is more practical to formulate theories which account for the fact that people can affect the probabilities they face. In fact, it seems that the open debate on the issue ended in the early seventies, and that both views now co-exist. For example, general equilibrium theory treats probabilities as given (Debreu, 1959, Arrow, 1970, Radner, 1982), while non-cooperative game theory, in the spirit of the games against nature formulated by Von Neumann and Morgenstern (1944), supposes that every player decides on the probability of his actions, and in this way determines the probabilities faced by others. Financial portfolio models treat probabilities as exogenous (Magill and Quinzii, 1997) but they are endogenous in the health economics of prevention (Zweifel and Breyer, 1997) and the logit and probit estimation in econometrics (Greene, 1997) treat them as dependent variables.

Risk versus insecurity

This paper adopts the insecurity perspective and explores the scope for incorporating the endogeneity of probability distributions within general equilibrium analysis. The following expected utility maximization problem may summarize Hirschleifer's objections to such an undertaking. Suppose that a decision maker maximizes the expected value of his utility $u(x;e)$, under the continuous density $f(e;x)$ with compact support A, where utility is continuous in the deterministic choice variable x, for all x in the compact convex set $C \subset R^n$, and integrable in the random variable e. If we assume that the maximum is attained, this expected utility maximization may be written:

$$U^* = max_{x \in C} \int_A u(x;e) f(e;x) de \,, \tag{1.1}$$

and clearly, reflects a problem of insecurity management, since the distribution of e is now taken to be under the decision maker's control. However, in line with Hirschleifer's argument, we can rephrase it into a risk management problem. For this, defining the uniform density $g(e)$ with the same support A as f, and the density weighted utility function $U(x;e) = u(x;e) f(e;x) / g(e)$, we obtain the risk management problem:

$$U^* = max_{x \in C} \int_A U(x;e)g(e)de \; , \qquad (1.2)$$

or, for discrete states, indexed s, with fixed probabilities P_s:

$$U^* = max_{x \in C} \sum_s P_s U(x;\boldsymbol{e}_s). \qquad (1.3)$$

Since $u(x;e)$ and $U(x;e)$ are continuous, they can both be thought of as being founded on the same basic axioms of choice under uncertainty (see Arrow, 1970). We also note that in either way, the variation in x induces a shift in the distribution of utility, illustrating that the distribution of the endogenous variables of a decision problem is itself endogenous, irrespective of whether the underlying density of the random shock is fixed or not. Because of this formal equivalence, we refer to (1.1) and (1.2) as using an insecurity and a risk format, respectively, since they adopt different formulations to represent the same economic reality.

The choice of a model format is generally guided by the application at hand. For example, computer simulation often uses the risk format, because it can be implemented easily by drawing for every element of the vector $\boldsymbol{e}$, a random number from the unit interval, and mapping these on the support A. However, a formal micro-economic analysis will require expected utility to reflect individual preferences, and find it hard to interpret the density weighted utility $U(x;e)$, that is a product of a utility and a density function. Furthermore, whenever the decision is to be embedded in an equilibrium model, concavity of the utility function becomes important, to avoid discontinuity of demand. Finally, the actions in the vector x should be interpretable and implementable. Specifically, if in the decision model in insecurity format the same density applies to several agents, it manifestly possesses the common public good characteristics of non-rivalry and non-excludability, and requires co-ordination mechanisms.

Overview

While insecurity management has become a popular area of study, particularly in the insurance literature[1], its integration within general equilibrium analysis and the associated welfare theory seems to be lacking. This is problematic, because it hides certain causes of potential market failure, specifically, non-convexity, non-rivalry and non-excludability, and hence for intervention. Section 2 considers the non-convexity, in a continuous as well as in a discrete probability setting, and section 3 the non-rivalry and non-excludability. Section 4 formulates a

[1] For example: Under the Safety Laws for 2001, a mandatory automobile insurance premium discount for safe, mature drivers is established. This requires insurance companies to provide an appropriate three year premium discount to drivers 55 years or older if they have completed a crash prevention course approved by the Department of Public Safety, Bureau of Highway Safety. (Source: Maine Bureau of insurance (2001). New York Central Mutual fire insurance company offers discounts on premiums if home owners implement specific measures to prevent fire. (Source: New York Central Mutual Fire Insurance Company (2002)).

general equilibrium model of a competitive economy, starting from the Arrow Debreu model with a complete set of contingent markets, and extending it to allow for endogenous variation in probability distributions. While it is easy to show that the non-convexity and the public good characteristic challenge the basic existence results as well as the Pareto-efficiency of the equilibrium, our main aim is to identify conditions where existence is preserved, and to look for institutional arrangements that can efficiently manage insecurity, specifically, the relatively vulnerable arrangements such as Lindahl pricing, in combination or as alternative to the more robust and decentralizable standard-based labeling.

As our main concern is decentralization, the treatment will remain within the confines of expected utility theory, and not address conditions of catastrophic risk, where events with dramatic consequences occur rarely, or affect few individuals only. It is well known that such situations are to be dealt with through government intervention, including criminal prosecution.

Section 2
Non-convexity

We differentiate between, on the one hand, conditions where the non-convexity seems inessential, because there is a wide class of specifications that do not suffer from it (section 2.1), and, on the other hand, conditions where it seems inescapable and calls for dedicated arrangements to maintain decentralization (section 2.2). The dividing line is essentially whether the utility function is independent of decisions taken at the beginning of the period, or not. Throughout, we assume that the utility and density functions are concave in decision variables, reflecting decreasing marginal effect of an intervention.

2.1 Utility independent of beginning-of-period decisions

Uncertainty introduces a time dimension. The expected utility maximization problem (1.1) is special in that all decisions are taken to be made before uncertainty is revealed. Clearly, decisions affecting the shape of the density function are necessarily in advance, and typically relate to prevention and promotion. Yet, practice is also characterized by cures, that can be expressed as end-of-period or coping actions, to be taken when an ailment has become manifest. It appears that the non-convexity disappears when decisions taken at the beginning of the period do not affect the end-of-period utility, or only do so in a specific, recursive manner.

Coping. Any coping decision, taken after uncertainty has been revealed, can be supposed to be subsumed in $u(x;e)$ via:

$$u(x;e) = max_{z \in Z(x,e)} u(z,x;e), \tag{2.1}$$

where $Z(x,e)$ is a nonempty, compact convex set. Therefore, in situations where u depends on z only, and Z on e only, the payoffs $u(e)$ are fixed, and the non-convexity vanishes from (1.1).

Expected value constraint. The separability is preserved if the original model is extended to include expected value constraints. This could be, for example, a budget constraint in which expected expenditure is incremented by the cost of changing the density, or a commodity balance for a market where all risk is idiosyncratic, i.e. where all states materialize simultaneously and the density is realized fully, as in a lottery (for ease of notation, we will drop the restriction A under the integral), and we impose non-negativity restriction on $x(e)$:

$$\begin{aligned} & max_{y \geq 0, x(e) \geq 0} \int u(x(e);e) f(e;y) de \\ & subject\ to \\ & g' y + \int k(e)' x(e) f(e;y) de \leq b, \qquad (l) \end{aligned} \tag{2.2}$$

where g is the price vector of inputs y, and where l denotes a Lagrange multiplier, and b is a constant such that a strict inequality is feasible on this constraint (Slater's constraint qualification). By defining the surplus, as the conjugate function:

$$s^*(lk;e) = max_{x\geq 0}\{u(x;e) - lk'x\}, \tag{2.3}$$

which is convex in its first argument(see e.g. Avriel, 1976, p. 106), it becomes possible to rewrite (2.2) in the minimax form in deterministic variables only:

$$min_{l\geq 0}\, max_{y\geq 0}[\int s^*(lk(e);e)f(e;y)de - lg'y]. \tag{2.4}$$

In the next section, we further describe the properties of this type of minimax problem.

Contingent constraints. The surplus (2.3) also permits to account for contingent, that is e-specific, density weighted constraints, that further restrict the coping decision, say, by commodity balances on contingent markets replacing the fixed prices $k(e)$ in the surplus by state-contingent prices:

$$s^*(p;e) = max_{x\geq 0}\, u(x;e) - p'(x - w(x;e)), \tag{2.5}$$

where the concave function $w(x;e)$ could represent a production function of state dependent endowments, where prices correspond to the constraint $x - w(x;e) \leq 0$, for which we assume, as before that it is feasible to let it hold with strict inequality. Next, we can use this surplus in the maximin problem:

$$max_{y\geq 0} \int min_{p(e)\geq 0}[s^*(p(e);e)f(e;y)de] - g'y, \tag{2.6}$$

to obtain the optimal distribution of market clearing contingent prices $p(e)$, as well as optimal inputs y.

Recursive decisions. One step further in allowing for simultaneity between the prevention and investment actions in the density and utility functions, respectively, is to specify a utility function $u(z(e), x; e)$, that has constant returns to scale in (z, x), where x is a scalar, while the density f is concave in (y/x) instead of y. This means that the actions (e.g. inputs) needed to control the density depend on the scale variable x. The decision problem becomes:

$$U^* = max_{(x,y)\in C} \int max_{z(e)}\, u(z(e), x; e) f(e; y/x)de. \tag{2.7}$$

To verify concavity, note that $\tilde{u}(e)x = max_{z(e)}u(z(e),x;e)$. Hence, concavity is ensured if it holds for the extended density function:[2] $\tilde{f}(e;x,y) = xf(e;y/x)$.

Representation with discrete probabilities

So far, we presumed that it was possible to specify continuous densities that are concave in the relevant decision variables. In practice, this may not be easy. The main difficulty in this connection is that the mean derivative $\int f'(x;e)de$ should be zero for all x, since the integral of the density should add up to unity. Common nonlinear forms such as the quadratic, semi-log or Cobb Douglas do not meet this requirement. Moreover, it is not transparent how a concave density could follow from micro-economic principles. We show that the representation becomes relatively straightforward in a setting with discrete probabilities, since it permits to view the control of probabilities as a regular production process.

Moreover, even though models with continuous densities are amenable to numerical implementation by means of stochastic optimization techniques − that can be invoked to solve equilibrium problems, and can also address non-convexities − (see Ermoliev et al., 2000), the representation with discrete probabilities is easier in its numerical implementation.

To make the transition to discrete probabilities, we return to risk format (1.3) with S uncertain states, indexed s, and write model (1.3) in insecurity format, with probability $P_s(x)$, as:

$$U^* = max_{x \in C} \sum_s P_s(x) u_s(x), \tag{2.8}$$

where we write $u_s(x) = \int u(x;e) f_s(e)de$, where f_s is the density of e associated to the realization of state s.

We represent the endogenous probabilities by extending the constraint set C of deterministic variables, to include P_s, which is now also deterministic, writing $(P,x) \in D$, where D, is taken to be compact convex, and to include the technology for controlling probabilities. For example, to represent prevention, we could specify the constraint set $D = \{(P,x) \mid \underline{P}_s(x) \le P_s \le \overline{P}_s(x), all\ s, \sum_s P_s = 1, x \in C\}$, for convex, non-negative prevention functions $\underline{P}_s(x)$, such that $\sum_s \underline{P}_s(x) < 1$ for all $x \in C$, and concave, non-negative promotion functions $\overline{P}_s(x)$, such that $\sum_s \overline{P}_s(x) > 1$ for all $x \in C$. This illustrates that it is not particularly difficult to maintain convexity when probabilities are discrete.

Let us briefly review the implications for the earlier specifications.

(a) *Coping*. This is readily dealt with, as (2.1) becomes:

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[2] For positive x and $(x,y) \in C \subset R^n_+$, concavity is ensured (see Ginsburgh and Keyzer, 1997, Theorem A.1.5) but in addition, we also need to postulate continuity of $\tilde{f}$ at $x=0$.

$$U^* = max_{(P,x)\in D} \sum_s P_s u_s \ , \tag{2.9}$$

for fixed $u_s = max_{z_s \in Z_s} u_s(z_s)$, which is obviously concave, since it has a linear objective, and a convex constraint set.

(b) *Expected value constraint.* Program (2.2) transforms into:

$$max_{(P,y)\in D, x_s \geq 0 \ all \ s} \{ \sum_s P_s u_s(x_s) \mid g'y + \sum_s P_s k_s' x_s \leq b \}. \tag{2.10}$$

Clearly, this is not a convex problem, since probability functions are multiplied with decisions x_s. Conversion to a convex problem can be performed along two lines. The first uses the minimax form, as in (2.4):

$$min_{l \geq 0} \ max_{(P,y)\in D} [\sum_s P_s s_s^*(l k_s) - l g' y] \ , \tag{2.11}$$

for a surplus function $s_s^*(l k_s) = max_{x_s} \{ u_s(x_s) - l k_s' x_s \}$, similar to (2.3). The second defines the probability weighted variables $X_s = P_s x_s$ and writes the decision in terms of X_s, P_s and y:

$$max_{(P,y)\in D, X_s \geq 0, all \ s} \{ \sum_s P_s u_s(X_s / P_s) \mid g'y + \sum_s k_s' X_s \leq b \}, \tag{2.12}$$

where we concavity of $P_s u_s(X_s / P_s)$ follows, for positive P_s (as in footnote 1), and the existence of a bounded Lagrange multiplier l is ensured, by Slater's constraint qualification.

(c) *Contingent constraints.* The model with contingent constraints becomes:

$$max_{(P,y)\in D, x_s \geq 0, all \ s} \{ \sum_s P_s u_s(x_s) - g'y \mid P_s x_s \leq P_s w_s(x_s), all \ s \}, \tag{2.13}$$

while supposing that $0 \leq x_s < w_s(x_s)$ is feasible for all *s*. This defines the equivalent of surplus function (2.5):

$$s_s^*(p_s) = max_{x_s \geq 0} u_s(x_s) - p_s'(x_s - w_s(x_s)) \tag{2.14}$$

which is convex non-increasing, and leads to the minimax problem:

$$min_{p_s \geq 0, all \ s} \ max_{(P,y)\in D} [\sum_s P_s s_s^*(p_s) - g'y] \ , \tag{2.15}$$

where the inner program is concave, and has a value function that is convex non-increasing in $(p_1,...,p_S)$. Alternatively, a convex program could be obtained by writing the model in probability weighted form, as in (2.12).

(d) *Recursive decisions.* Recall that x is a scalar in this case, and that the probability function depends on the ratio y/x:

$$U^* = max_{(x,y)\in C} \sum_s P_s(y/x)u_s^*(x). \quad (2.16)$$

An equivalent convex problem, is obtained if we define the constraint set

$$D=\{(X,x,y)\,|\,x(\underline{K}_s-\underline{g}_s(y/x))\leq X_s\leq x(\overline{K}_s+\overline{g}_s(y/x)), all\ s; \sum_s X_s = x; (x,y)\in C\}, \quad (2.17)$$

where concavity of $x\underline{g}_s(y/x)$ and $x\overline{g}_s(y/x)$ follows, for positive x, along the same lines as for the expected value constraint, and program

$$U^* = max_{(X,x,y)\in D} \sum_s X_s\tilde{u}_s^*, \quad (2.18)$$

with given $\tilde{u}_s^* = max_{z_s} u_s(z_s,1)$, is convex.

Existence of saddlepoints of minimax problems

We have seen in (2.11) and (2.12) that the problem with endogenous probabilities can be dealt with either via conversion to a convex program with probabilities and probability weighted actions as decision variables, or via a minimax (or maximin) problem. However, both are not equivalent. On the one hand, as is well known, any convex program also defines a pair of minimax and maximin problems, that if the program satisfies Slater's constraint qualification, possesses a saddlepoint. On the other hand, the minimax formulation of a non-convex program does not always possess a saddlepoint.

In our context, the existence of a saddlepoint is important for two reasons. First, there is in problems such as (2.11) a basic arbitrariness in choosing between a minimax and a maximin formulation. Consider the function $f: X\subset R^m\times Q\subset R^n\to R, f(x,q)$. A point $(\overline{x},\overline{q})$ with $\overline{x}\in X$ and $\overline{q}\in Q$ is a saddlepoint if:

$$f(\overline{x},q)\leq f(\overline{x},\overline{q})\leq f(x,\overline{q}). \quad (2.19)$$

If such a saddlepoint exists (see e.g. Avriel, 1976, p. 55), both problems reach the same value for the objective (no duality gap):

$$min_{q\in Q}\, max_{x\in X}\; f(x,q) = max_{x\in X}\, min_{q\in Q}\; f(x,q), \tag{2.20}$$

eliminating the arbitrariness. Second, let Q^* and X^* denote the optimal sets of the outer optimization problems on the left and the right hand side, respectively, and let $X^\circ(q^*)$, $Q^\circ(x^*)$ denote the optimal sets for some $q^*\in Q^*$ and $x^*\in X^*$. Then, (2.20) implies that

$$X^\circ(q^*) = X^*$$

and (2.21)

$$Q^\circ(x^*) = Q^*$$

Consequently, if a saddlepoint exists, the set of optimal values of both the left and the right hand side problems is $X^*\times Q^*$, and whenever f is concave in x and convex in q, and the sets X and Q are convex, it follows that $X^*\times Q^*$ is convex. Since this is the case for the minimax problem associated to convex program (2.12), the minimax problem associated to the original, non-convex problem has a saddlepoint as well. Consequently, there is no arbitrariness in writing a minimax or maximin in (2.11) and since its constraint sets are convex, its solution set is convex. The convexity property is essential to prove existence of equilibrium when the minimax problem has parameters that are determined in a larger model.

Coping behavior in a multi-period context

So far, we only considered a two-period setting with decisions at the beginning and the end of the period. It appears that the coping model extends to multiple periods if probabilities are independent between periods. For example, with a discount factor b, a three-period version of the coping model (2.9), with probabilities P_s^0 at the intermediate time point, and of the realization t of P_t^s conditional on both the realization of and the prevention measures taken in state s can be written in the recursive form:

$$U^* = max_{(P^0,y_0)\in D}\,[\textstyle\sum_s P_s^0 u_s^* - p_0' y_0 + b\sum_s P_s^0\, max_{(P^s,y_{1s})\in D}[\sum_t P_t^s u_t^* - p_{1s}' y_{1s}]]. \tag{2.22}$$

Thus, this form lends itself to a backward recursion, without non-convexity. At the same time, it illustrates that the interdependence between the probabilities over time can be an additional cause of non-convexity, either because the states are stochastically dependent, or because the effect of prevention measures depends on inputs in earlier periods. In (2.22), this would be reflected in the dependence of the constraint set D in the inner optimization at state s, on P^0 and y_0.

We conclude that, in the two-period expected utility framework, the loss of concavity is primarily due to investment effects in utility, and that in the absence of these effects, ensuring concavity is cumbersome if densities are continuous, but relatively straightforward if they are discrete. However, when the dynamics become more involved, with lagged effects of inputs and consumption spreading over several periods, the non-convexity becomes almost inescapable.

2.2 Standards and labels

To stress the basic distinction between discrete probabilities and discretization of the probability profiles, we briefly return to continuous densities to introduce standardization of profiles.

Standards. Discretization is a common way of dealing with non-convexity. In an institutional sense, it amounts to letting the central planner evaluate different standards, and select the most promising one. In terms of (1.1), if the vector x partitions into (x_1, x_2) and the density only depends on x_2, we can discretize the action space into different standards indexed r, on the basis of associated fixed actions x_2^r, $r = 1,...,R$, and density functions $f(e; x_2^r)$. This supposes that standards do not reduce uncertainty by themselves, unlike Jones and Hudson (1996), but the modification is easily made. Decision model (1.1) now becomes restricted to:

$$U^* = max_r \; max_{(x_1, x_2^r) \in C} \int u(x_1, x_2^r; e) f(e; x_2^r) de \;, \tag{2.23}$$

in which all non-convexity has been eliminated from the inner maximization problem. The equivalent with discrete probabilities s, is immediate:

$$U* = max_r \; max_{(x_1, x_2^r) \in C} \sum_s P_s(x_2^r) u_s(x_1, x_2^r). \tag{2.24}$$

Clearly, the optimum of the discretized problem will generally lie below that of the original, non-convex one, and the finer the discretization, the smaller the distance will be.

Labels. In many applications, decision problems are to be embedded within an equilibrium model, that also determines some of their parameters, say, market prices. Then, non-convexities may cause inefficiency and even non-existence of equilibrium because of the discontinuity of the behavioral response to parameter changes but the discrete choice of standards is not an option, since it also causes discontinuity. To maintain continuity, convex combinations of discrete solutions should be admitted. For a continuous density this leads to

$$U^* = max_{w_r \geq 0, \sum_r w_r = 1} \sum_r w_r \; max_{(x_1, x_2^r) \in C} \int u(x_1, x_2^r; e) f(e; x_2^r) de \;, \tag{2.25}$$

while the discrete probability version is:

$$U^* = max_{w_r \geq 0, \sum_r w_r = 1} \sum_r w_r \; max_{(x_1, x_2^r) \in C} \sum_s P_s(x_2^r) u_s(x_1, x_2^r). \tag{2.26}$$

We note that, even though the optimum of the problems taken in isolation will tend to specialize with $w_r = 1$, this may not be so for an equilibrium solution – with parameters g depending on w and x_1 and appearing as arguments in utility and density functions – and in this case, the interpretation of the outcome might be problematic, as this optimal convex combination might be distant from the true optimum in (1.1), and not reflect a real possibility. In section 4, we discuss situations where this discretization-plus-convexification has a clear institutional interpretation as a labeling device. We can already see here that convexification arises by enabling individuals freely to compose their basket from a given set of probability profiles or standard-based labels. This adds variety to discretization, in contrast to both the single standard solution (2.22) and (2.23) and the original situation (1.1), in which they only face a single probability profile. In addition, the solution with labeling cannot yield lower utility than with discrete standard, and, for a sufficiently fine grid of points x_2^r, it will come close to the global optimum of (1.1).

Section 3
Security as a public good

In the sphere of demand for knowledge related public goods, it has been argued recently (e.g. Boldrin and Levine, 2002) that pure non-excludability and non-rivalry are rare, as most processes, such as the replication of knowledge, take time, and often also space. Yet, it appears that when different agents are confronted with the same controllable uncertainty, security becomes a pure public good. For example, with respect to the probability of suffering a particular illness, of falling victim to a crime, or of becoming unemployed, there often is no way of excluding anyone from falling under the density (non-excludability).

Non-rivalry is also inescapable. For I individuals indexed i, who choose deterministic consumption x_i, with a common density that depends on the collective action y, we can interpret the utility function $u(x;e) = \sum_i a_i u_i(x_i;e)$ as a social welfare function with given weights a_i:

$$U^* = max_{(x_1,...,x_I,y)\in C} \sum_i a_i \int u_i(x_i;e) f(e;y) de \ , \qquad (3.1)$$

where y is the non-rival decision. Hence, the process controlling the density is typically public. Unlike non-convexity, this public element does not pose any problem for the model formulation itself but it points to difficulties of co-ordination in actual practice.

These well known co-ordination problems can be dealt with either by imposing a non co-operative structure whereby every agent takes the behavior of other (strategic) agents as given, which avoids the problem, or by designing an institution through which agents can communicate so as to agree on the collective action.

Non-cooperative game. In a non co-operative setting, individual i may have to choose an optimal strategy x_i, given the decisions x_{-i} of the other individuals.

$$U_i(x_{-i}) = max_{x_i \in C_i} \int u_i(x_i, x_{-i}; e) f_i(e; x_{-i}) de \ . \qquad (3.2)$$

This circumvents both the non-convexity and the non-rivalry problem, but it will not yield efficient solutions.

Lindahl pricing. Alternatively, one may seek to develop an institution that can determine the optimal density, say, by means of Lindahl pricing. For example, in the model with contingent constraints, the utility function could, like in (3.1) be viewed as a social welfare function and its surplus (2.5) decomposed into the individual surpluses

$$s_i^*(p;e) = a_i u_i(x_i;e) - p'(x_i - w_i(e)) . \qquad (3.3)$$

From this, the agents willingness to pay f_i could be determined, as the derivative of

$$W_i(y) = \int s_i^*(p(e);e)f(e;y)de \ , \qquad (3.4)$$

and conversely the individually optimal level y_i can be determined at given prices f_i, from

$$W_i^*(f_i) = max_{y_i} \int s_i^*(p(e);e)f(e;y_i)de - f_i y_i \ , \qquad (3.5)$$

and these prices can be adjusted until the optimal levels coincide across individuals. Clearly, all usual remarks with respect to free riding apply.

Section 4
Embedding in general equilibrium

We use the discrete probabilities to apply the insecurity format within a general equilibrium model. In a general equilibrium model of pure exchange that fits within the Arrow Debreu framework, we include endogenous probabilities, and explore the conditions under which the existence and efficiency are preserved under the usual assumptions.

4.1 Insecurity management under pure exchange: weather shocks on contingent markets

We initially consider aggregate risk, say, due to climatic variability, or plagues. This affects all markets and leads to state specific prices p_s, in contrast to the idiosyncratic risk, to be dealt with in the next section, where uncertainty only affects individuals, and all states are realized simultaneously, like in a lottery.

We start from model (2.13) with contingent constraints, and to close the economy, introduce in addition, a commodity balance for the beginning of the period. Activities x refer to consumption, while y is an input demand that controls probabilities. All consumption is ex post and all consumers face the same probabilities.. The utility, endowment and prevention/promotion functions possess the following properties.

Assumption C1 (utility): (i) Utility functions are strictly concave and homogeneous; (ii) they are non-satiated for $i \neq 1$, and strictly increasing for $i = 1$; (iii) endowment functions are concave and nonnegative; and (iv) positive for $i = 1$; (v) $\underline{P}_s(y)$ is strictly convex and, $\overline{P}_s(y)$ is strictly concave, $\sum_s \underline{P}_s(y) < 1$ and $\sum_s \overline{P}_s(y) > 1$ for all non-negative y.

We can construct these bounds on probabilities for the prevention of hazards and the creation of opportunities via ordinary production processes, according to:

$$\begin{aligned} \underline{P}_s(y) &= \underline{K}_s - \underline{g}_s(y) \\ \overline{P}_s(y) &= \overline{K}_s + \overline{g}_s(y) \end{aligned} \tag{4.1}$$

for $s \neq 1$, where $\sum_{s\neq 1} \underline{K}_s < 1$, $\sum_{s\neq 1} \overline{K}_s < 1$, $\underline{g}_s : R^n_+ \to [0, \underline{K}_s)$ and $\overline{g}_s : R^n_+ \to R_+$ are strictly concave, homogeneous production functions. This highlights that in this formulation with discrete probabilities the control of insecurity can be dealt with as for an ordinary production process.

We can now formulate a competitive general equilibrium model that defines an equilibrium between a rent maximizing producer decision for insecurity management, and a consumer specific decision that chooses demand, and communicates a willingness to pay for better security. The producer decision is:

$$\begin{aligned} P \;=\; & \max_{y\geq 0, P_s \geq 0, all\ s} \textstyle\sum_s P_s \sum_i s_{is} - p_0{}' y \\ & subject\ to \\ & \quad \underline{P}_s(y) \leq P_s \leq \overline{P}_s(y) \qquad s \neq 1 \\ & \quad \textstyle\sum_s P_s = 1, \end{aligned} \tag{4.2}$$

where this profit is equal expected (money metric) value of the end-of-period consumer surplus minus the cost of inputs, indicating that this is the criterion of a public agency. Every consumer solves:

$$\begin{aligned} & \max_{x_{i0}\geq 0, x_{is}\geq 0, P_{is}\geq 0\ all\ s} u_{i0}(x_{i0}) + \textstyle\sum_s P_{is} u_{is}(x_{is}) \\ & subject\ to \\ & \quad p_0{}' x_{i0} + \textstyle\sum_s P_{is} p_s{}' x_{is} = p_0{}' w_{i0}(x_{i0}) + \sum_s P_{is} p_s{}' w_{is}(x_{is}) + t_i \qquad (l_i) \\ & \quad P_{is} = P_s, \qquad (\tilde{s}_{is}) \end{aligned} \tag{4.3}$$

for net transfer consisting of a share in the rent $t_i = q_i P - x_i$, where $x_i = \sum_s P_s s_{is}$ is the access price to the lottery equal to the expected value of the surplus. Equilibrium is found for commodity prices p_0, p_s such that the markets clear:

$$\begin{aligned} & \textstyle\sum_i x_{i0} + y \leq \sum_i w_{i0}(x_{i0}) \\ & \textstyle\sum_i x_{is} \leq \sum_i w_{is}(x_{is}), \end{aligned} \tag{4.4}$$

and, finally, the money metric consumer surplus is communicated to the agency:

$$s_{is} = \tilde{s}_{is} / l_i. \tag{4.5}$$

Hence, it is not evident that the public good nature of probability fosters solidarity. On the one hand, where different groups fall under a common probability distribution, those with a high surplus (the rich) in a particular state will be willing to contribute more to reaching favorable states and avoiding distress. On the other hand, since the profit shares are exogenous, the net payments of the rich might not be higher.

For fixed input y in (4.2), model (4.2)-(4.4) reduces to a standard Arrow-Debreu model with contingent markets, as in Debreu (1959). The equilibrium will exist, but it will only be Pareto-efficient conditional on this fixed value, hence the need to determine an efficient level. We also note that in (4.3) only expected utility has only to be concave and non-satiated. Thus, we might allow for decreasing utility in some states, so as to represent unfavorable events, for which no financial compensation is possible. However, we also observe that all commodity balances have free disposal, as expressed by the inequality as opposed to the equality constraint. This preserves non-negativity of all equilibrium prices, since it supposes that consumers always have the possibility not to buy a good that does not provide utility. In section 4.5 below, we discuss the implication of dropping the possibility of free disposal.

Alternatively, we could also formulate a model in which consumers, rather than communicating their surpluses, seek agreement on probabilities:

$$\max_{x_{i0} \geq 0,\, x_{is} \geq 0, P_{is} \geq 0 \text{ all } s} u_{i0}(x_{i0}) + \sum_s P_{is} u_{is}(x_{is})$$
$$\text{subject to} \tag{4.6}$$
$$p_0' x_{i0} + \sum_s P_{is}(p_s' x_{is} + s_{is}) = p_0' w_{i0}(x_{i0}) + \sum_s P_{is} p_s' w_{is}(x_{is}) + q_i P \,, \qquad (l_i)$$

where surpluses act as Lindahl prices and are adjusted until the desired probabilities agree:

$$P_{is} = P_s\,. \tag{4.7}$$

Hence, we can consider two versions of this equilibrium model: (4.2)-(4.5) with communication on surpluses, and (4.2),(4.4), (4.6)-(4.7) with communication on desired probabilities. Clearly, in the second version, program (4.6) has a non-convexity because of the multiplication by probabilities, and in both versions efficiency of equilibrium is not obvious.

To verify existence and efficiency of equilibrium, we formulate the associated welfare program with the welfare weights α_i can be adjusted on the simplex, as in Negishi (1960), until the budget deficit b_i vanishes for every group *i:*

$$U^* = \max_{x_{i0} \geq 0, \text{all } i;\, y \geq 0, x_{is} \geq 0 \text{ all } i,s, P_s \geq 0} \sum_i \alpha_i [u_{i0}(x_{i0}) + \sum_s P_s u_{is}(x_{is})]$$
$$\text{subject to}$$
$$\sum_i x_{i0} + y \leq \sum_i w_{i0}(x_{i0}) \qquad (p_0)$$
$$P_s \sum_i x_{is} \leq P_s \sum_i w_{is}(x_{is}) \qquad (p_s) \tag{4.8}$$
$$\sum_s P_s = 1$$
$$\underline{P}_s(y) \leq P_s \leq \overline{P}_s(y).$$

The associated budget deficit is:

$$b_i = p_0'(x_{i0} - w_{i0}(x_{i0})) + \sum_s P_s p_s'(x_{is} - w_{is}(x_{is})) - q_i P \,, \tag{4.9}$$

for

$$\boldsymbol{P} = \sum_s P_s \sum_i \boldsymbol{s}^*_{is}(\boldsymbol{\alpha}_i, p_s) - p_0' y \tag{4.10a}$$

and surplus function:

$$s^*_{is}(a_i, p_s) = max_{x_{is} \geq 0}\, a_i u_{is}(x_{is}) - p_s'(x_{is} - w_{is}(x_{is})). \tag{4.10b}$$

We note that homogeneity of utility and nonnegativity of endowments in assumption C1 imply that the surplus is nonnegative for every state. Yet, since we did not require utility to be increasing in all states it might happen that optimal consumption is zero.

Assumption C2 (constraint qualification): Slater's constraint qualification holds, i.e. there exists in (4.8) a feasible allocation with strict inequalities for all constraints.

We can now prove existence and efficiency of equilibrium, remarking that the non-standard aspect to be tackled is that program (4.8) is not convex. Whereas the probabilities in the constraints only appear to obtain interpretable multipliers, and could be dispensed of the product forms with P_s in the objective cannot be eliminated.

Proposition 1 (existence and efficiency of equilibrium): If assumptions C1 and C2 hold, then (a) there exists a Negishi equilibrium (4.8)-(4.10); (b) every such equilibrium is also an equilibrium of the Arrow Debreu model with contingent markets, and endogenous insecurity both in the version (4.2)-(4.5) with communication on surpluses and in version (4.2),(4.4), (4.6)-(4.7) with communication on desired probabilities; (c) it is Pareto efficient.

Proof.

Part a. We can write (4.8) in the probability weighted consumptions $X_{is} = P_s x_{is}$, which leads to a convex program that satisfies Slater's constraint qualification. We can also write it in the minimax form:

$$U^* = min_{p_s \geq 0, all\ s}\ max_{(P,y,x_{10},\ldots,x_{I0}) \in D} \sum_i [a_i u_{i0}(x_{i0}) + \sum_s P_s s^*_{is}(a_i, p_s)], \tag{4.11a}$$

for $D = \{(P, y, x_{10},\ldots,x_{I0}) \mid \underline{P}_s(y) \leq P_s \leq \overline{P}_s(y), all\ s, \sum_s P_s = 1, \sum_i x_{i0} + y \leq \sum_i w_{i0}(x_{i0})\}$.

with deficits:

$$b_i = p_0'(x_{i0} - w_{i0}(x_{i0})) + \sum_s P_s b_{is} - q_i P \tag{4.11b}$$

for

$$b_{is} = a_i \frac{\partial s^*_{is}}{\partial a_i} - s^*_{is} = a_i u_{is} - s^*_{is}, \tag{4.11c}$$

and

$$\boldsymbol{P} = \sum_s P_s \sum_i s^*_{is}(a_i, p_s) - p_0' y, \tag{4.11d}$$

where, by Assumption C1, differentiability follows from strict concavity of utility and endowments in (4.10b). Existence of bounded multipliers follows from C2. As usual in the Negishi format, the equilibrium is obtained as the fixed point of a mapping obtained from a program and associated budget deficits, by including an adjustment rule for welfare weights that raises the weights of groups with a budget surplus, and scales all weights to keep them on the simplex (see e.g. Ginsburgh and Keyzer, 1997, chapter 3). We only refer to steps where the argument differs from the standard Negishi proof. The special point is that problem (4.11a) is a minimax problem, instead of the usual convex welfare program. As it is derived from a non-convex program it might, as discussed in section 2, exhibit a duality gap and have no saddlepoint, even though its objective is concave in quantities $(P,y,x_{10},...,x_{I0})$ and convex in prices $(p_1,...,p_S)$, and its constraints are convex. Hence, the main step is to prove that the gap vanishes.

Denoting the compact domains and ranges of variables x by *(x)*, we construct the fixed point mapping by defining, first the correspondence from welfare weights on the simplex, to probabilities, initial consumption and prices in (4.11a): $(a) \Rightarrow (P,y,x_{10},...,x_{I0},p_1,...,p_S)$, second, the continuous function from welfare weights, prices, prevention input and consumption in (4.11b-d) and the adjustment rule: $(a,p,y,x_{10},...,x_{I0}) \rightarrow (a)$. Now the uppersemicontinuity (usc) of this correspondence is immediate, since the optimal choice mapping of both minimum and the maximum problem are usc, by the maximum theorem, and the minimax problem generates a composition of both. However, its convex-valuedness is less obvious as the set of optimal quantities $(P,y,x_{10},...,x_{I0})$ might depend on optimal prices.

Now, by the argument in (2.21), to every optimal price corresponds the same quantity set, because these prices are also Lagrange multipliers in the convex program with probability weighted quantities. This ensures that a saddlepoint exists and that program (4.11a) defines a correspondence $(a) \Rightarrow (P,y,x_{10},...,x_{I0}) \times (p_1,...,p_S)$, whose range, the Cartesian product of the price and the quantity set, is convex.

Existence of a fixed point now follows via the Kakutani theorem. And in equilibrium all prices are positive, and the welfare weight is positive for group 1, because the utility function of group 1 is increasing, and this group has positive endowments.

Part b. With $l_i = 1/a_i$, the consumer problem and the optimal decision directly follow from the Lagrangean of (4.10), and the commodity balances appear explicitly in (4.10), whose the multipliers are the equilibrium prices.

Part c. Since all utility functions are non-satiated, and the utility functions of group 1 are increasing, and group 1 has a positive welfare weight, welfare optimality implies Pareto-efficiency. ■

4.2 Idiosyncratic risk: employability

In the model of the previous section, the economy faced aggregate risk, and all individuals were defending themselves against a common external threat, while seeking to make optimal use of external opportunities. Under such conditions, the need to co-operate may seem obvious. In the present section, we consider the opposite situation, of idiosyncratic risk, whereby the probabilities materialize fully within the period under consideration. They may refer to the probability of falling ill of a particular disease *s*, or of reaching a specified occupation *s*, possibly unemployment. In modelling terms, the difference from the previous section is that all realizations appear on the same commodity balance, that becomes an expected value constraint, as in (2.10), and consolidate the s-specific commodity balances in (4.4) into:

$$\sum_s P_s \sum_i x_{is} \leq \sum_s P_s \sum_i w_{is}(x_{is}) \qquad (p_1) \tag{4.12}$$

with a single market price p_1, and write the minimax form (4.11a):

$$U^* = \min_{p_1 \geq 0} \max_{(P, y, x_{10}, \ldots, x_{I0}) \in D} \sum_i [a_i u_{i0}(x_{i0}) + \sum_s P_s s^*_{is}(a_i, p_1)] \,. \tag{4.13}$$

The decentralization is precisely as in the earlier section, except that $p_s = p_1$ has to be imposed. Specifically, individuals face lotteries and need to agree on the use of inputs used to improve the probabilities of, say, being spared an infectious disease, receiving timely medical treatment, being admitted to a job.

Alternatively, the following representation with probability weighted consumption $X_{is} = P_s x_{is}$, treats probability P_s as well as X_{is} as commodities:

$$U^* = \max_{x_{i0} \geq 0,\, all\ i;\, y \geq 0,\, X_{is} \geq 0\, all\ i,s,\, P_s \geq 0} \sum_i a_i [u_{i0}(x_{i0}) + \sum_s P_s u_{is}(X_{is} / P_s)]$$

subject to

$$\begin{aligned} &\sum_i x_{i0} + y \leq \sum_i w_{i0}(x_{i0}) && (p_0) \\ &\sum_s \sum_i X_{is} \leq \sum_s P_s \sum_i w_{is}(X_{is} / P_s) && (p_1) \\ &\sum_s P_s = 1 \\ &\underline{P}_s(y) \leq P_s \leq \overline{P}_s(y), \end{aligned} \tag{4.8}$$

which is a regular Negishi program in probability weighted quantities. This illustrates that any general equilibrium model can accommodate endogenous probability.

4.3 Production

Next, we extend the previous model to include production, more explicitly than with the endowment functions introduced so far. It would be straightforward to replace these by transformation functions but the essential step is to represent the dependence of output in state s on initial inputs, and the recursive formulation (2.15) makes it possible to do this with preservation of convexity.

$$
\begin{aligned}
q_{k0} &= f_{k0}(v_{k0}) \\
q_{ks} &= f_{ks}(v_{ks}, q_{k0}),
\end{aligned}
\tag{4.14}
$$

for $k = 1,...,K$, where the output q_{ks} of commodity k depends on the input vector v_{ks}, and the production level q_{k0} just before uncertainty is revealed. For a crop, this level could be the dry matter production, say, one month after planting. Then, it would be natural to suppose that the eventual yield per unit of this dry matter depends on the inputs v_{ks} applied to it per unit, and hence to postulate that f_{ks} has constant returns to scale. This production enters the commodity balances (4.4) as:

$$
\begin{aligned}
&\Sigma_i x_{i0} + y + \Sigma_k v_{k0} \le \Sigma_i w_{i0} + \Sigma_k q_{k0} \\
&\Sigma_i x_{is} + \Sigma_k v_{ks} \le \Sigma_i w_{is} + \Sigma_k q_{ks},
\end{aligned}
\tag{4.15}
$$

and the producer maximizes expected profits

$$
\boldsymbol{P}_k = (p_{k0} + \Sigma_s P_{ks} \boldsymbol{p}_{ks}(p_{ks}, p_s)) q_{k0} - p_0' v_{k0},
\tag{4.16a}
$$

subject to (4.14), for profit

$$
p_{ks}(p_{ks}, p_s) q_{k0} = max_{v_{ks} \ge 0} [p_{ks} f_{ks}(v_{ks}, q_{k0}) - p_s' v_s]
\tag{4.16b}
$$

and $P_{ks} = P_s$, and this term adds to the objective of the minimax problem (4.11a). The difficulty is now that the probability entering the expected unit profit in (4.16a) is endogenous, and multiplies with q_{k0}, causing a non-convexity, that also makes the decision problem (4.2) of the prevention agency non-convex. This non-convexity can be made to vanish if the probability is product specific, independent of P_s, and if its management, unlike P_s, requires scale dependent inputs, like in the recursive decisions (2.16)-(2.17), with probability constraints:

$$\begin{aligned}&\underline{K}_{ks} - \underline{g}_{ks}(y_k / q_{k0}) \leq P_{ks} \leq \overline{K}_{ks} + \overline{g}_{ks}(y_k / q_{k0}), \; s \neq 1 \\ &\textstyle\sum_s P_{ks} = 1,\end{aligned} \qquad (4.17)$$

for concave production functions $\underline{g}_{ks}$ and $\overline{g}_{ks}$. If we now define the probability weighted initial output, $Q_{ks} = P_{ks} q_{k0}$, we obtain, for every k, the convex program:

$$\begin{aligned}&P_k^* = \max_{v_{k0} \geq 0, q_{k0} \geq 0; Q_{ks} \geq 0, \text{all } s} \; p_{k0} q_{k0} + \textstyle\sum_s Q_{ks} p_{ks}(p_{ks}, p_s) - p_0' v_{k0} \\ &\textit{subject to} \\ &\quad \underline{K}_{ks} q_{k0} - \underline{g}_{ks}(y_k / q_{k0}) q_{k0} \leq Q_{ks} \leq \overline{K}_{ks} q_{k0} + \overline{g}_{ks}(y_k / q_{k0}) q_{k0}, \; s \neq 1 \\ &\quad \textstyle\sum_s Q_{ks} = q_{k0} \\ &\quad q_{k0} \leq f_{k0}(v_{k0})\end{aligned} \qquad (4.18)$$

which can readily be inserted in the general equilibrium model of Proposition 1, after specifying a distribution of profits q_{ik} among consumer groups.

Thus, it appears that the potential market failure arises if the production risk is dependent on the input decisions at the beginning of the period (area planted) and the prevention cannot be undertaken for every product separately.

4.4 Taxation and insurance

The endogeneity of probabilities also has significant implications for taxation, essentially because what appears like a Pareto-efficient lump sum transfer in a setting with exogenous probabilities, may become distortionary once individual agents recognize the endogeneity. In particular, consumer problem (4.3) shows that any (discounted) transfer t_{is}, contingent on reaching a particular state will affect the Lindahl prices in the program:

$$\begin{aligned}&\max_{x_{i0} \geq 0, x_{is} \geq 0, P_{is} \geq 0 \text{ all } s} u_{i0}(x_{i0}) + \textstyle\sum_s P_{is} u_{is}(x_{is}) \\ &\textit{subject to} \\ &\quad p_0' x_{i0} + x_{i0} + \textstyle\sum_s P_{is} p_s' x_{is} = p_0' w_{i0}(x_{i0}) + \textstyle\sum_s P_{is}(p_s' w_{is}(x_{is}) + t_{is}) - f_i' y \qquad (l_i) \\ &\quad P_{is} = P_s, \qquad (s_{is})\end{aligned} \qquad (4.19)$$

for given initial fee $\boldsymbol{x}_{i0} = \sum_s P_s \boldsymbol{t}_{is}$, and thus cause a distortion. It seems obvious that this happens if the insured person has a possibility to affect probabilities. However, program (4.12) shows that it will also be the case if this person takes probabilities as given but can affect prevention. Clearly, no such distortion would arise in a model with fixed probabilities. It would also vanish if the fixed indemnity schedule with fixed t_{is} was replaced by self-insurance, with t_{is} endogenously determined as the financial deficit of every state, that is as the optimal value of:

$$max_{x_{i0}\geq 0, x_{i0}; x_{is}\geq 0, P_{is}\geq 0, t_{is}\ all\ s}\ u_{i0}(x_{i0}) + \sum_s P_{is} u_{is}(x_{is})$$

subject to

$$\begin{aligned} & p_0' x_{i0} + x_{i0} = p_0' w_{i0}(x_{i0}) - f_i' y && (l_i) \\ & p_s' x_{is} = p_s' w_{is}(x_{is}) + t_{is} && \\ & x_{i0} = \sum_s P_{is} t_{is} && \\ & P_{is} = P_s, && (\tilde{s}_{is}) \end{aligned} \qquad (4.20)$$

noting that (4.12) can be written in the same way, but with t_{is} and x_{i0} fixed. Clearly, the distortion would also disappear if t_{is} was equal across states.

In fact, there is another interpretation, whereby the transfers t_{is} are illegal, and robbers receive them as income, while honest people pay. The balance of revenue and expenditures is only imposed for $\sum_i x_{i0} = \sum_i \sum_s P_s t_{is}$ in total, rather than for every consumer group separately, and with t_{is} as a function of endogenous variables, but taken as given by individuals.

4.5 Standards

The model of section 4.1 supposes full separability between beginning and end-of-period. Its utility functions separate as $U_i = u_{i0}(x_{i0}) + \sum_s P_s u_{is}(x_{is})$, and its endowment functions only depend on the consumption in the same period. A more general formulation drops the separability and writes $U_i = \sum_s P_s u_{is}(x_{i0}, x_{is})$ for expected utility, and This would allow to reflect for example, the uncertain effect of an investment in prevention of epidemics. At this point, it also may be relevant to make explicit that every population group i actually consists of N_i identical individuals. Now for assumption C1 applying to Negishi program (4.1) and consumer problem (4.3) modified in this way, we can define the surplus:

$$\begin{aligned} & s^*_{is}(a_i, p_0, p_s) = \\ & \quad max_{x_{i0}\geq 0, x_{is}\geq 0}\ a_i u_{is}(x_{i0}, x_{is}) - p_0'(x_{i0} - w_{i0}(x_{i0})) - p_s'(x_{is} - w_{is}(x_{i0}, x_{is})) \end{aligned} \qquad (4.21)$$

and the minimax problem (4.11a) accordingly into:

$$U^* = min_{p_0\geq 0, p_s\geq 0, all\ s}\ max_{(P,y)\in D}\ [\sum_i N_i \sum_s P_s s^*_{is}(a_i, p_0, p_s) - p_0' y] \qquad (4.22a)$$

for $\quad D = \{ P \mid \underline{P}_s(y) \leq P_s \leq \overline{P}_s(y), all\ s, \sum_s P_s = 1 \},$ (4.22b) where the remaining non-convexity is due to the product form of probabilities and consumption dependent surpluses, and we consider the option of standardization to deal with it.

Under standardization, there is some authority that restricts the possible probability profiles $\hat{P}_{rs}$, implicit in D, to a finite number R of options indexed r, from which a selection is to be made by collective choice. Associated to option r are the fixed input $\hat{y}_r$. Basically, this amounts to solving the model of section 4.1, for every profile, which requires obtaining Arrow-Debreu equilibria with fixed probabilities, and in some way choose the best one, and it obviously raises the question of defining a criterion for selection. If lumpsum income transfers are possible, one might adopt a bargaining approach, and define a welfare program, with a common welfare criterion across profiles, and possibly a common set of lower bounds on expected individual utilities, to reflect reservation. But clearly, this treatment of aggregate risk requires central planning, as it does not eliminate the non-convexity.

4.6 Labeling

The alternative is standard-based labeling. If aggregate risk prevails, it is often impossible to manage insecurity in this way, essentially because the shocks may affect all people simultaneously. Yet, one could imagine that the population decides to spread the risk by dividing itself into subgroups with different risk profiles. For example, to control the damage from diluvial rains, some might build boats and rafts, while others migrate to the highlands. Hence, if it is possible to let various profiles co-exist, i.e. to combine standards with variety, labeling becomes an option. And even though the non-rivalry and collective choice persist, the non-excludability may be eliminated by assigning a single label to every individual.

To reflect this collective choice, we suppose that all individuals face the same probability w_r of falling under profile r, but we will see that group-specific probabilities with choices w_{ir} are readily dealt with as well. In this section, the probabilities may be thought of as referring to the aspects of morbidity and illiteracy, that depend on investments in collective facilities. We maintain the formulation for aggregate risk, re-iterating that idiosyncratic risk can be represented by consolidation of commodity balances across s, while $p_s = p_1$. The label is chosen so as to maximize the rent:

$$
\begin{aligned}
&P = max_{w_r \geq 0,\, all\ r} \sum_r w_r (\sum_i y_{ir} N_i - p_0{}' \hat{y}_r) \\
&subject\ to \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4.23) \\
&\quad \sum_r w_r = 1.
\end{aligned}
$$

while the individual consumer takes this choice of label as given and solves:

$$\max_{x_{ir0}\geq 0,\, x_{irs}\geq 0, w_{ir}\geq 0 \text{ all } r,s} \sum_r w_{ir} \sum_s \hat{P}_{rs} u_{is}(x_{ir0}, x_{irs})$$

subject to (4.24)

$$\sum_r w_{ir}(p_0' x_{ir0} + \sum_s \hat{P}_{rs} p_s' x_{irs}) = \sum_r w_{ir}(p_0' w_{i0}(x_{ir0}) + \sum_s \hat{P}_{rs} p_s' w_{is}(x_{ir0}, x_{irs})) + t_i \qquad (l_i)$$

$$w_{ir} = w_r, \qquad (\tilde{y}_{ir})$$

for given net transfer $t_i = q_i P / N_i - x_i$, equal to the income from rent minus the cost of the lottery ticket, $x_i = \sum_r w_r y_{ir}$, where $y_{ir} = \tilde{y}_{ir} / l_i$ is the money metric willingness to pay for label *r*. Commodity balances (4.4) are imposed as before. The equivalent concave program can be obtained in terms of the weighted consumption $X_{ir0} = w_{ir} x_{ir0}$ and $X_{irs} = w_{ir} x_{irs}$:[3]

$$\max_{X_{ir0}\geq 0,\, X_{irs}\geq 0, w_{ir}\geq 0 \text{ all } r,s} \sum_r w_{ir} \sum_s \hat{P}_{rs} u_{is}(X_{ir0}/w_{ir}, X_{irs}/w_{ir})$$

subject to (4.25)

$$\sum_r p_0' X_{ir0} + \sum_s \hat{P}_{rs} p_s' X_{irs} =$$

$$\sum_r w_{ir}[p_0' w_{i0}(X_{ir0}/w_{ir}) + \sum_s \hat{P}_{rs} p_s' w_{is}(X_{ir0}/w_{ir}, X_{irs}/w_{ir})] - t_i \qquad (l_i)$$

$$w_{ir} = w_r. \qquad (\tilde{y}_{ir})$$

Here the choice of label is imposed, but as in (4.6), the consumer would support this choice if confronted with this Lindahl price:

$$\max_{x_{ir0}\geq 0,\, x_{irs}\geq 0, w_{ir}\geq 0 \text{ all } r,s} \sum_r w_{ir} \sum_s \hat{P}_{rs} u_{is}(x_{ir0}, x_{irs})$$

subject to (4.26)

$$\sum_r w_{ir}(p_0' x_{ir0} + \sum_s \hat{P}_{rs} p_s' x_{irs} + y_{ir})$$

$$= \sum_r w_{ir}(p_0' w_{i0}(x_{ir0}) + \sum_s \hat{P}_{rs} p_s' w_{is}(x_{ir0}, x_{irs})) + q_i P / N_i \qquad (l_i)$$

The formulation is easily adapted to allow for group-specific choice of option w_{ir}, that makes it possible to escape the co-ordination problems of public good provision. We also note that via the endowment functions, individuals can invest in their labor productivity by themselves, via the consumption input x_{ir0} in the endowment functions, but this will not affect the odds they face. An intermediate generalization would differentiate probabilities by group, and represent the combined effect of rival and non-rival inputs:

---

[3] Concavity of the objective follows as in footnote 1, if $nu_{is}(X_0/n, X_s/n)$ is continuous at $n = 0$, for every *i*, *s*, which is a mild requirement.

$$D_i = \{ P_i \mid \underline{P}_{is}(y, y_i) \leq P_{is} \leq \overline{P}_{is}(y, y_i), all\ s, \sum_s P_{is} = 1 \}, \quad (4.27)$$

which would make it possible to account for group specific risks and interventions. This formulation leads to a group-specific choice of label w_{ir}, but with a non-rival input constraint: $y_{0k} \geq \sum_r w_{ir} k_{irk} \hat{y}_{irk}$.

Checking existence and efficiency of equilibrium can as in section 4.1 proceed via the Negishi program, that transforms to:

$$U^* = \max_{x_{ir0} \geq 0, all\ i; x_{irs} \geq 0, N_{ir} \geq 0, w_r \geq 0, all\ i,r,s} \sum_i \alpha_i \sum_r N_{ir} [\sum_s \hat{P}_{rs} u_{is}(x_{ir0}, x_{irs})]$$

subject to

$$\begin{aligned}
&\sum_i \sum_r N_{ir} x_{ir0} + \sum_r w_r \hat{y}_r \leq \sum_i \sum_r N_{ir} w_{i0}(x_{ir0}) && (p_0) \\
&\sum_i \sum_r N_{ir} \hat{P}_{rs} x_{irs} \leq \sum_i \sum_r N_{ir} \hat{P}_{rs} w_{irs}(x_{ir0}, x_{irs}) && (p_s) \\
&\sum_r w_r = 1 \\
&N_{ir} = w_r N_i && (y_{ir})
\end{aligned} \quad (4.28)$$

with budget deficits:

$$b_i = \sum_r w_r b_{ir} \quad (4.29a)$$

for $b_{ir} = p_0'(x_{ir0} - w_{i0}(x_{ir0})) + \sum_s \hat{P}_{rs} p_s'(x_{irs} - w_{is}(x_{ir0}, x_{irs})) + (y_{ir} - q_i \boldsymbol{P} / N_i)$,
where

$$\boldsymbol{P} = \sum_r w_r (\sum_i \boldsymbol{y}_{ir} N_i - p_0' \hat{y}_r). \quad (4.29b)$$

This program can be written in a form that has a concave objective for the maximization in the minimax problem, by defining the population weighted initial consumption:

$$\min_{p_0 \geq 0, p_s \geq 0, all\ s}$$

$$\max_{w_r \geq 0, N_{ir} \geq 0, all\ i,r} \sum_r \sum_i N_{ir} \sum_s \hat{P}_{rs} s^*_{is}(\alpha_i, p_0, p_s) - p_0' \sum_r w_r \hat{y}_r$$

subject to (4.30)

$$\begin{aligned}
&\sum_r w_r = 1 \\
&N_{ir} = w_r N_i && (y_{ir})
\end{aligned}$$

and the associated budget deficit becomes:

$$b_i = \sum_r w_r b_{ir} \tag{4.31}$$

for $b_{ir} = \sum_s \hat{P}_{rs} b_{is} + y_{ir} - q_i P / N_i$, and $b_{is} = a_i \frac{\partial s^*_{is}}{\partial a_i} - s^*_{is} = a_i u_{is} - s^*_{is}$, and labeling rent P defined as in (4.29b).

Product labels

So far, our discussion attributed risk profiles to individuals but the model could be reinterpreted to relate them to commodities. Yet, if product safety is to be accounted for, it becomes necessary to drop the assumption of free disposal, since the problem is precisely that the consumer buys a commodity bundle of an uncertain composition with a fixed probability profile, and cannot dispense of less favorable outcomes. Hence, marginal utility may be negative in some state s, and cause prices and incomes to be negative. In the welfare programs this is to be reflected via equality constraints for commodity balances, while in the minimax problems, the price variables become unconstrained in sign. The standard will now refer to a product label. Since there is no forced purchase of any labeled commodity, the labeled product will have a price equal to the expected price of characteristics, $f_r = \sum_s \hat{P}_{rs} p_s$, that is necessarily non-negative. In this case, the choice of label w_{ir} can be looked at as a commodity in itself, with price r_i measuring the rent on the constraint $\sum_r w_{ir} = 1$, and every group i can be taken to buy a bundle (w_i, X_i), at prices (r_i, f), by maximizing its utility function:

$$U(w_i, X_i) = \sum_r U_{ir}(w_{ir}, X_{ir}), \tag{4.32}$$

for $U_{ir}(w_{ir}, X_{ir}) = w_{ir} \sum_s \hat{P}_{rs} u_{is}(X_{ir} / w_{ir})$. Clearly, in practical applications the label will have to be differentiated by commodity.

Section 5
Conclusion

We have described a variety of situations in which agents face the same probability distribution, that can be affected by them collectively, by containing health risks, promoting employability, and enforcing public safety. Through this, these distributions acquire public good characteristics, and make collective choice inevitable. Furthermore, in a context that distinguishes decisions at the beginning and the end of the period, the management of insecurity faces potential market failure if individual utility or production depend on purchases at the beginning of the period. In such situations, labeling of risk profiles, possibly of commodities, offers a cure because it provides the individual with an opportunity to choose. It appears that these cases are readily incorporated in general equilibrium models. Through this, it for example, becomes possible to reinterpret existing models of endogenous growth (Romer, 1990) that link investments in non-rival goods to long term growth as a theory links security management to growth.

We conclude with a remark on the Rawlsian veil of ignorance. Under the Rawlsian construct, individuals are supposed to choose the social institutions on the basis of the expectation that they will be in the worst position. Hence, they choose so as to maximize the minimal utility. Consequently, all conflicts of interests disappear among participants. Arrow (1977) presented a well known criticism of this construct, pointing to the fact that it implies extreme risk aversion, and indicating that even under perfect ignorance, expected utility maximization would lead to the individual and collective welfare function $W(x) = \sum_i P_i u_i(x_i)$, for given probability P_i, equal across *i*. This would allow for inequality of realized utilities u_i .Hence, even under this construct, the veil of ignorance permits to circumvent Arrow's own impossibility theorem. However, in Arrow's framework, social choice is purely concerned with finding a criterion for choosing between alternative configurations *x* and x'. This is problematic in that it supposes that all what happens to individuals fully depends on institutions, without making explicit what these actually entail. In fact, Arrow criticized the veil-of-ignorance construct in that it is not possible to decide about anything under perfect ignorance of all institutions. Here we have proposed to treat the probabilities themselves as the institutions. Then, social choice becomes a decision as how to shape the odds, i.e. the probability of individuals to end up in a particular situation, and the optimal set of institutions is determined as the ideal probability configuration, while income transfers follow as insurance premiums. Specifically, in terms of the model with contingent markets, and a single consumer, optimal probabilities are the solution of the welfare program:

$$max_{(P,X)\in D} \sum_s P_s u_s(X_s / P_s) \qquad (6.1)$$

for

$$D = \{(P,X) \mid \underline{P}_s(y) \le P_s \le \overline{P}_s(y), all\ s, \sum_s P_s = 1, x_s \ge 0, (y + \sum_s X_s) \in C\}, \qquad (6.2)$$

highlighting that, if empathy is sufficient, the barriers to collective choice dissipate.

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Centre for World Food Studies
SOW-VU
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

Telephone (31) 20 - 44 49321
Telefax (31) 20 - 44 49325
Email pm@sow.econ.vu.nl
www http://www.sow.econ.vu.nl/