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Food safety, labeling and market efficiency

by

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Contents

Abstract

The safety of a given food item may be expressed as a product characteristic that is uncertain at the time of transaction. It can be represented through a probability distribution. Improving food this safety amounts to changing the shape of the distribution, through prevention measures. Extreme risks will lead to prohibition, but if financial compensation of losses is conceivable, the situation becomes more complex, as economic agents have to decide simultaneously on quantities and probabilities, for example in the calculation of expected utility and expected profits. This creates a non-convexity that may cause market failure, even in situations where all safety characteristics are priced competitively. The paper follows two lines of investigation to avoid this problem. The first considers restrictions on the functional forms of utility and prevention functions, and the second introduces the institution of standard-based labeling as a filter on possible distributions, essentially to discretize them. Starting from a partial equilibrium model with one consumer, one producer and one unsafe commodity, in which various options prove effective, we gradually extend the analysis to cover several agents and commodities, eventually arriving at a general equilibrium formulation. At every step, we eliminate unsuitable options, eventually to find labeling as sole viable option.

Section 1
Introduction[1]

In the wake of the crises around BSE and foot and mouth disease, food safety has become a prominent item on the policy agenda of the EU and other developed countries. In reaction, various control mechanisms are currently being put in place, especially to monitor and to certify food quality and food safety. Minimum standards are being raised and the demands on product labeling tightened. In addition, the range of consumer concerns has been expanding in recent years and nowadays extends to animal friendliness, labor standards, and environmental sustainability (Caswell, 1998), strengthening the need for adequately certified labels. At present, one part of the debates evolve around the need for public intervention to manage this process, as opposed to letting the industry develop adequate practices on its own (see e.g. Henson and Caswell, 1999 for an overview of issues), another addresses the desirability of international harmonization of food safety standards. Developing countries have, in particular, questioned whether the food safety concerns are genuine or only serve as new variants of old non-tariff barriers, now that the classical tariff barriers have increasingly come under attack (OECD, 1999).

The theoretical issue is, therefore, why commodities should, for other reasons than protectionism, obey pre-specified standards. A label offers the consumer a description both in a physical and a moral sense of what he is buying. In addition, certified labels provide some guarantee that what is written on the label covers the relevant aspects of product safety, is understandable to the layman, and true. Through this it reduces the asymmetry of information between buyer and seller, to deal with the hold-up problem inherent in the delay between purchase and consumption, and avoid the prevalence of 'lemons' (Akerlof, 1970), whereby the market only supplies inferior qualities. One further explanation been that standards facilitate R&D, save on information costs, and make it possible for producers to reap the returns to scale in design, and the network externalities (see Katz and Shapiro, 1994, and Matutes and Regibeau, 1996).

However, while it seems obvious that efficiency is improved when information asymmetries are eliminated at low cost and that there may be gains from co-ordination when different parts of a product chain or a network agree, it must be noted that standards go a step further and restrict the choice to at most a predefined list of discrete alternatives, while standard-based labels permit several standards to co-exist. Indeed, it appears that standards also function inside organizations, where information asymmetries are limited. Moreover, the arguments for free international trade and elimination of quantitative restrictions generally rely on first-best arguments, that abstract from these asymmetries. In fact, classical models of product differentiation even allow for a continuum of commodities (Lancaster 1966, 1991), or a

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[1] The author thanks Ferdinand Pavel and Lia van Wesenbeeck for comments.

continuum of product varieties (Dixit and Stiglitz, 1977). In such models, discretization would cause a loss of welfare.

Yet, it is well known that restrictions can be essential for market efficiency. For example, budgets and more generally, the obligation to pay for delivered goods, impose restrictions on individual behavior that actually form the very bedding of decentralization and hence of market efficiency (North, 1990). However, these budget restrictions are taken to be frictionless and supposedly implemented via harsh but costless punishments, of the type described by Becker (1968). Standards and standard-based labels on their part, necessarily restrict the space of feasible allocations.

In this paper, we argue that standards and standard-based label can be interpreted as devices to avoid market failures. Indeed, if product safety can be affected by economic agents, these failures become imminent, even if all agents possess perfect information on all product characteristics.

First, since the probability distribution of safety characteristics becomes endogenous, expected profit and expected utility calculations suffer from an inherent non-convexity. By means of a numerical example we show that this may keep the economy trapped at a low-level equilibrium. Second, since it is generally impossible to supply every consumer with a tailormade bundle of characteristics that suits him most, the choice of the optimal safety profile becomes collective, and through this, probabilities acquire the public good characteristics for all agents that face them, and lead to the common market failures, unless a dedicated co-ordination mechanism is put in place. Third, it appears that in an efficient solution, the profits from prevention cannot be charged to consumers. Consequently, the prevention agency must recover them from producers directly, which implies vertical integration and may threaten competition. Finally, when product unsafety prevails, the markets for safety characteristics have no free disposal, since the consumer has to buy all characteristics before the actual quality is revealed. We show in a general equilibrium model, that this may lead to negative prices, and hence to bankruptcy and market failure. All these issues would remain hidden if the analysis distinguished a finite number of productqualities, described by a product label, and treated standards as a mere instrument for reducing the information asymmetry and for banning the poorest qualities.

In seeking to avoid the market failures, we proceed along two avenues. The first applies separability restrictions on the functional forms of the utility and prevention functions in seeking to eliminate the non-convexity, and we find that cannot be circumvented easily. The second adopts standard-based labeling, with special reference to the discretization aspect. Discretization of options is a common means to facilitate the search for optimality if non-convexity prevails. Inn a centrally planned system it permites to select the best policy option or standard. Standard-based labeling extends this capability to a market economy, essentially because it enables consumers to buy several labeled varieties at the same time, hence eliminating all non-convexities. This also weakens the need for vertical integration while the co-ordination requirements of collective choice are alleviated as well, since agreement is only needed a list of effective standards rather than on a full probability profile.

Since the paper concentrates on deriving conditions under which a first-best solution can be achieved, it does not pay attention to the extensive literature on the effects of second-best policy measures on welfare and the behavior of consumers and producers. Contributions in this field include, for example, Buzby et al. (1998) who make a cost-benefit analysis of food safety risk reductions; Buzby and Frenzen (1999) on the effects of criminal lawsuits on producer incentives to provide safe food; and more generally, Benson (1998) on the possible benefits of privatization of crime prevention, and Broder and Morrall (1991) on general incentives for firms to act safely.

1.2 Overview

The paper proceeds as follows. In section 2, we specify a partial equilibrium model of consumption and retail with a single consumer and a single commodity. We analyze this model in section 3, illustrating by means of a numerical example, that an inefficiency can occur, that we avoid via restrictions on functional forms and via labeling. Next, we expand the model, in section 4, by allowing for production, and in section 5, for heterogeneous commodities and consumers. Finally, in section 6 we make a transition from partial to general equilibrium, and find that none of the restrictions on functional forms can rule out bankruptcy, and are, therefore, left with labeling as sole remaining option.

Section 2
Prevention in a partial equilibrium model of consumption and retail

We distinguish *S* uncertain states, indexed *s*, to represent the hazard associated to the purchase of a quantity, occurring with probability P_s. Prevention affects these probabilities, using quantities *e* of prevention inputs per unit of consumption.

Whereas Hirschleifer (1970) has claimed that it is always possible to define states independently from human actions, in our case prevention, we follow Ehrlich and Becker (1972, p. 638), who argued that this is not a tenable proposition. Clearly, if we were to distinguish states by prevention measure as well as by random event, say, $s=1$: no fire, $s=2$: fire with an extinguisher available in the room, and $s=3$: fire without an extinguisher available, we would implicitly discretize the options and, besides suffering a curse of dimensionality, neglect that the real world process at hand might be continuous rather than discrete, which, as will be seen, may precisely be a cause of market failure.

We initially suppose that there is a single commodity, a consumer good with raw material price p^c – superscript *c* for crude. A social planner, a consumer who also decides on prevention, buys this good in quantity *x*, and applies a quantity of prevention inputs *e*. In this partial equilibrium setting, we take the price of the prevention input *e* to be equal to unity and measured in the same unit as utility (see e.g. Willig, 1976). To introduce the non-convexity issue, we present a simple example.

Example 1 Suppose that there are two states, $s=1$ for safe, or high quality food, and $s=2$ for unsafe or low quality food. The probability of the second state is affected by prevention, requiring input *d* to reduce the probability of the second state by d^{β}, $P_2 = 1 - P_1 = max(\hat{P}_2 - d^{\beta}, 0)$; and $u_s(x) = x^{\gamma_s}$, where coefficients β and γ_s lie between zero and unity, ensuring strict concavity of both the prevention and the utility function. Now the consumer's expected utility can, for P_2 less than unity be written $\sum_{s=1}^{2} P_s u_s(x) = x^{\gamma_1} + (\hat{P}_2 - d^{\beta})(x^{\gamma_2} - x^{\gamma_1})$.The second term is not concave.◊[2]

We note that concavity could be restored in this case if prevention was scale dependent i.e. obeyed $(d/x)^{\beta}$, and the utility function had a separable gain: $u_2(x) = u_1(x) - x^{\eta_2}$. Hence, we initially consider scale dependent prevention functions and make specific assumptions about the gain.

Next, we formulate the planning problem in general terms, supposing that it attains its optimum, as the surplus maximization:

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[2] In fact, in the second term we have a product of two concave expressions, which is pseudoconcave, but the sum of the concave first term and the pseudoconcave second term is not necessarily pseudoconcave.

$$\begin{aligned}
&max_{P_s \geq 0, all\ s; e, x \geq 0} \sum_s P_s u_s(x) - (p^c + e)x \\
&subject\ \ to \\
&P_s \geq \overline{P}_s(e),\ s \neq 1 \\
&\sum_s P_s = 1.
\end{aligned} \tag{2.1}$$

Here also the endogeneity of P_s and e may cause the program to be non-convex, under the following assumptions on utility and prevention:

Assumption U1 (utility): (a) For every s, the utility functions $u_s : R_+ \to R$ are strictly concave and twice differentiable, with uniformly bounded first derivatives; (b) for any positive *x*, the safe state, $s = 1$, is superior: $f_s(x) = u_1(x) - u_s(x) > 0$.◊

This assumption is weak in that it does not require utility to increase with consumption in every state, or even to be positive. Imposing stronger requirements would be questionable because there is no opportunity for free disposal of consumption in separate states. One might question whether concavity can be imposed for these separate states, since consumer theory only derives this property on the basis of revealed preference considerations, which only apply for expected utility over all states. The requirement will be relaxed when we allow for labeling in Assumption U4 below. Here we maintain it because we want to make the point that non-convexities can emerge when utility functions are well behaved.

We also observe that the continuity of utility functions already rules out catastrophic risk. If the unsafety is such that it yields utility of minus infinity, for any $s \neq 1$, the planner will have to either ban the product, or, if this yields finite utility, shift the probabilities of all these states to their minimum level, irrespective of prices and preferences and let the consumer choose whether he wants to buy the product. Consequently, all probabilities become exogenous and the non-convexity is eliminated. Furthermore, we assume that the prevention technology separates as:

$$\overline{P}_s(e) = \hat{P}_s - G_s(e),\ s \neq 1, \tag{2.2}$$

where $\hat{P}_s > 0$ denote the maximal probability – if it was zero, this particular state could be dropped – and $\sum_{s \neq 1} \hat{P}_s < 1$. The prevention functions $G_s(\cdot)$ are ordinary production functions with decreasing returns to scale.

Assumption P1 (prevention): The scalar prevention functions $G_s : R \to [0, \hat{P}_s)$, defined for all $s \neq 1$, are homogeneous, continuously differentiable, strictly concave, and increasing.◊

The range of the prevention functions is limited to the open interval, to represent that prevention cannot fully eliminate all risk.

Equilibrium

The partial equilibrium model associated with the social plan (2.1) is, as usual in partial equilibrium, obtained by decomposition into a surplus maximization by the consumer and a profit maximization by the retailer. The consumer takes both price and probability as given, and maximizes his expected surplus at given consumer price p^f (*f* for finished as opposed to *c* for crude). The special feature is that the retailer decides on prevention *e*, and through this on probabilities P_s, given the consumer's valuation of safety characteristics p_s, rather than the consumer price p^f.

To highlight the nonrivalry of consumption across states we distinguish, within the consumer problem, consumption x_s by state, and retrieve the associated prices of characteristics, retrieved as the (possibly negative) Lagrange multipliers of the constraints $P_s x_s = P_s x$, that act as Lindahl prices:

$$\begin{aligned} &max_{x \geq 0; x_s \geq 0} \textstyle\sum_s P_s u_s(x_s) - p^f x \\ &subject\ \ to \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.3a) \\ &P_s x_s = P_s x, \qquad\qquad (p_s) \end{aligned}$$

where p_s is the price of safety characteristic *s*, that measures the consumer's willingness to pay and satisfies $p^f = \sum_s P_s p_s$. The retailer on his part is supposed to know these prices, and to take them as given, when maximizing his unit profit:

$$\begin{aligned} &max_{P_s \geq 0, all\ s; e \geq 0} \textstyle\sum_s P_s p_s - (p^c + e) \\ &subject\ \ to \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.3b) \\ &P_s \geq \overline{P}_s(e),\ s \neq 1 \qquad\qquad (\psi_s) \\ &\textstyle\sum_s P_s = 1, \qquad\qquad (\rho) \end{aligned}$$

and the consumer price adjusts until all profit is eliminated.

$$p^f = p^c + e. \qquad (2.3c)$$

Equilibrium is found if prices p_s of safety characteristics faced by the retailer coincide in (2.3b) with the consumer's willingness to pay in (2.3a). Finally, we need an assumption to keep consumption positive and bounded.

Assumption U2: (a) Positive consumption is desired: $\sum_s \hat{P}_s u_s'(0) > p^c$; (b) welfare is bounded: for $\bar{x}$ large enough, $max_s\, u_s'(\bar{x}) < p^c$. ◊

We can now prove:

Proposition 1 (existence of equilibrium) *Let assumptions P1, U1 and U2 hold. Then, model (2.3) has an equilibrium. Every equilibrium has positive consumption, zero profits, is a stationary point of the non-convex welfare program (2.1), and equilibrium prices are the Lagrange multipliers of this program.*

Proof.

(a) Consumption in (2.3a) is bounded. Since $x = 0$ and $x_s = 0$ can be implemented, the program is feasible. As $\sum_s P_s u_s'(x) \le max_s\, u_s'(x)$, by Assumption U2(b), expected marginal utility eventually drops below p^c and the program is bounded.

(b) Prices $(p_1,...,p_S)$ lie in the bounded set $Q \subset R^S$. This is implied by assumption U1(a).

(c) Equilibrium point exists. Let the simplex $\wp = \{(P_1,...,P_S) \in R_+^S \mid \sum_s P_s = 1\}$ denote the probability set. We construct a fixed point mapping of the compact convex set $\wp \times Q$ into itself, where program (2.3a) defines the correspondence $\wp \Rightarrow Q$, from probabilities to prices, and (2.3b) $Q \Rightarrow \wp$ from prices to probabilities. By Kakutani's fixed point theorem, a fixed point exists, and is obviously an equilibrium of (2.3a)-(2.3b), from which (2.3c) follows.

(d) Equilibrium consumption is positive. If $\sum_s P_s u_s'(0) = +\infty$, in equilibrium, zero consumption cannot be optimal. If it is bounded, it must be that $u_s'(0) < u_1'(0)$ for all $s \ne 1$. The first-order condition of (2.3a) w.r.t. x requires that $\sum_s P_s\, u_s'(x) - p^f \le 0$. Since $G_s(e)$ is increasing, assumption U2(a) implies that $\sum_s P_s u_s'(0) \ge \sum_s \hat{P}_s u_s'(0) > p^c$ for all $e \ge 0$. Hence, $x = 0$ cannot be an equilibrium.

(e) Equilibrium is stationary point of (2.1) To verify that an equilibrium is a stationary point of the welfare program, we rewrite (2.1) with an explicit commodity balance for characteristics as well as for raw material, after multiplication of the probability-constraint by output level *y*, and associate the Lagrange multipliers to the constraints (in brackets on the right-hand side):

$$\max_{P_s, x_s \geq 0, \text{all } s; e, x, y \geq 0} \sum_s P_s u_s(x_s) - (p^c + e)y$$

$$\text{subject to}$$

(a)

$$P_s x_s = P_s x \qquad (p_s)$$
$$x = y \qquad (p^f)$$
$$P_s y \geq \overline{P}_s(e)y,\ s \neq 1 \qquad (\psi_s)$$
$$\sum_s P_s = 1,$$

For equilibrium prices as candidate Lagrange multipliers, which by (c), are known to exist and to be bounded, we can write the Lagrangean of this nonconvex program as:

$$\max_{e,x,y \geq 0, P_s, x_s \geq 0, \text{all } s} \{ \sum_s P_s u_s(x_s) - \sum_s P_s p_s(x_s - x) + p^f(x - y) - (p^c + e)y$$
$$\sum_{s \neq 1} \psi_s (P_s y - \overline{P}_s(e)y) - \rho(\sum_s P_s - 1)\}.$$

We verify that the equilibrium defines a stationary point of this Lagrangean. Optimization w.r.t. y confirms (2.3c), while optimization w.r.t. x, and x_s yields (2.3a) and w.r.t. e gives (2.3b). Therefore, the equilibrium is a stationary point of (2.1) and conversely, since all constraints of (a) are met, (2.1) has bounded Lagrange multipliers.

(e) Zero profit. Finally, because of (2.3c), program (2.3b) yields zero profit. □

This equilibrium between the retailer and the consumer only establishes existence of a saddlepoint of the Lagrangean of the nonconvex program (2.1). Nonetheless, it may not be a global optimum, and inefficiency may result. We also mention two further difficulties

Valuing prevention

First, Lindahl prices p_s do not coincide with the rewards of the prevention activity itself, that are expressed as unit values ψ_s , and are, in accordance with Lagrangean in the proof, resulting from the profit maximizing decision:

$$\pi = \max_{e \geq 0} \sum_{s \neq 1} \psi_s G_s(e) - e, \qquad (2.4)$$

since $\min_{e \geq 0} \sum_{s \neq 1} \psi_s \overline{P}_s(e) + e$ yields the same optimal input demand e as (2.3b). Moreover, in (b), optimization w.r.t. P_s yields, at given $x = y$ and e, and if we substitute $u_1(x) - f_s(x)$ for $u_s(x)$:

$$\max_{P_s \geq 0, \text{all } s} \{ \sum_{s \neq 1} P_s(\psi_s x - f_s(x)) | \sum_s P_s = 1 \}. \qquad (2.5)$$

Therefore, for positive x we have

$$\psi_s = f_s(x)/x, \tag{2.6}$$

since $P_s > 0$ by assumption P1. Thus, the price of prevention is equal to the average increase of utility. We also note that in (2.3b), $p_s = \rho - \psi_s$ for $s \neq 1$, and $p_s = \rho$ for $s = 1$. Hence, $\psi_s = p_1 - p_s$ for all $s \neq 1$, indicating that the value of prevention is also equal to the difference between the prices of characteristics. Concavity and homogeneity of the prevention function ensure that the profit π will be positive, whenever inputs are purchased. The difficulty is now that it will not be possible for the agent engaged in prevention to recover this profit from the consumer directly, since, by (2.3c) the margin between consumer and raw material price only includes the current cost *e*.

Secondly, as usual in a model with uncertainty, since the demand exercised before uncertainty is revealed, it is non-rival across states. As the consumer takes both probabilities and the price p^f as given, and since we abstract from any hold-up problem, the retailer has to communicate with the consumer independently of any purchases, to find out about the prices p_s. If such communication is impossible, there will be no prevention inputs applied, probabilities will be given, only the market for quantities will equilibrate, at the price p^c, as in the market for 'lemons' (Akerlof, 1970), and, clearly, the outcome will be inefficient. We show in the next section that labeling can ease this communication problem. However, to highlight its more fundamental role, we assume perfect information and show that even when all characteristics are priced adequately, inefficiencies may arise, that can be ruled out via stringent restrictions on functional forms, and via labeling.

Section 3
Inefficiency and sufficient conditions for efficiency

3.1 Construction of an inefficient equilibrium

To highlight the possibility of inefficiency, despite the full information and the perfect pricing of characteristics, we construct an inefficient equilibrium. First, we define the welfare function as the composite mapping:

$$W(x) = V(u_1(x)/x,...,u_S(x)/x)x - p^c x \tag{3.1a}$$

where V is the convex value function:

$$V(\tilde{u}_1,...,\tilde{u}_S) = max_{P_s \geq 0,\, all\ s; e \geq 0}\{\sum_s P_s \tilde{u}_s - e \mid P_s \geq \hat{P}_s - G_s(e); \sum_s P_s = 1\}, \tag{3.1b}$$

for $\tilde{u}(x) = u(x)/x$. We note that V is a regular profit function, and, by Hotelling's lemma, $V_s' = P_s$, which is positive (by assumption P1) and adds up to unity. Hence, the welfare function has first derivative (in vector notation):

$$W'(x) = \tilde{u}'^T V' x + V - p^c, \tag{3.2}$$

and second derivative

$$W''(x) = (\tilde{u}'^T V'' \tilde{u}')x + V'^T u'', \tag{3.3}$$

because $u'' = x\tilde{u}'' + 2\tilde{u}'$. The first term is non-negative (by convexity of V), while the second is negative. Hence, the lack of concavity seems to be structural, with two obvious exceptions. The first term vanishes if either V is piecewise linear ($V'' = 0$), but this is excluded by strict concavity in P1, or if average utility is fixed ($\tilde{u}' = 0$). Below we point to some further exceptions but here we show that relationship (3.3) permits to construct examples with multiple stationary points.

Example 2

As a local approximation of an arbitrary function, we specify the quadratic form: $V(\tilde{u}) = b^T\tilde{u} + \frac{1}{2}\tilde{u}^T B\tilde{u}$ for $b \geq 0, b^T\iota = 1, B = L^T L$, where L is a lower triangular matrix such that $L^T\iota = 0$, which ensures positive semi-definiteness of $V'' = B$ and hence convexity of V. Now

$V' = b + B\tilde{u} = P$, and $V'^{T} \iota = 1$, while defining a range $[\underline{\beta}, \overline{\beta}]$ for $\tilde{u}$ where V' remains non-negative. Next, the utility function is specified as a quadratic concave approximation $\tilde{u}(x) = u(x)/x = \frac{1}{x}\alpha + a - \frac{1}{2}xA$, with positive coefficients, and $u' = a - xA$ and $u'' = -A$, while $\tilde{u}' = -\frac{1}{x^2}\alpha - \frac{1}{2}A$.

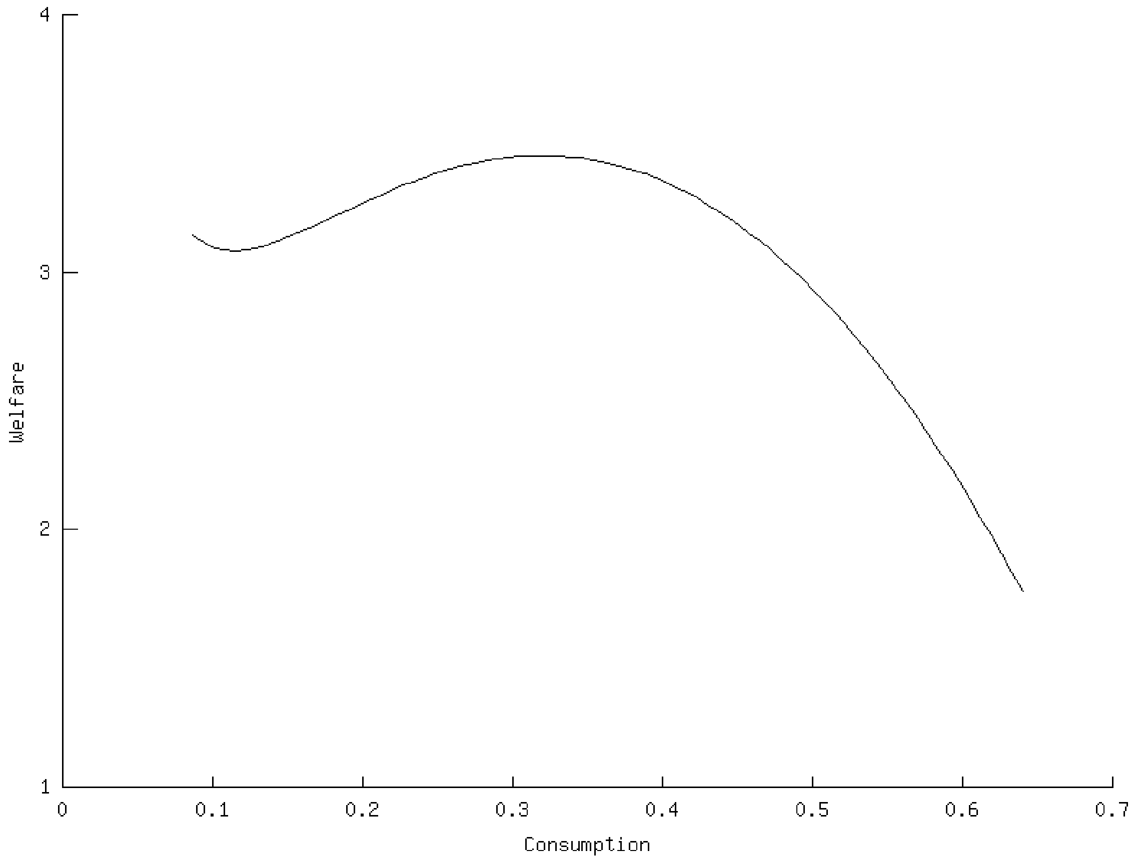
Values of *x* cannot be taken arbitrarily small, since $\tilde{u}$ has to be kept within $[\underline{\beta}, \overline{\beta}]$. To generate a convex segment in the welfare function, we define the constant $c_s = a_s - \frac{1}{2}A_s x$, and a fraction λ. This fraction reduces α, *x* and *A* by the same multiplication factor, and associated positive factors μ_s to scale a_s : $\mu_s = (c_s + \frac{\lambda^2}{2}xA_s)/a_s$, so as to keep $\tilde{u}(x)$ constant. Then, the second-order derivative can, for small λ be made arbitrarily large.

$$W''(x;\lambda) = \left((\frac{1}{x^2}\alpha + \frac{1}{2}A)^T B(\frac{1}{x^2}\alpha + \frac{1}{2}A)x \right) / \lambda + \left(V'^{T} A\right)\lambda .$$

Eventually, with rising *x*, the first term vanishes and $W''(x;\lambda)$ becomes negative, as illustrated in Figure 1, for coefficient values given in the Appendix. We also note that the scaling by the factor λ only affects the general unit of measurement of utility, which is not directly observable at any rate, and the linear coefficient a_s, which is scaled by μ_s. Hence, the utility function is adjusted according to $u_s(x) = \lambda(\alpha_s + (\frac{c_s}{\lambda} + \frac{\lambda}{2}A_s\hat{x})x - \frac{1}{2}A_s x^2)$, where $\hat{x}$ denotes the point at which the function was evaluated initially. ◊

Figure 1 shows a welfare program with at least two stationary points, and Proposition 1 that every equilibrium is a stationary point of (2.1). In addition, since the solution of (2.3a) and (2.3b) is fully characterized by the first-order conditions, we can assert that every stationary point, including in Figure 1 the local minimum of the welfare function obtained after substituting out all constraints in program (2.1), is an equilibrium.

We have seen that every for arbitrary concave utility and convex profit function, associated to a concave prevention function, we can construct quadratic approximations. We scaled the coefficient of the utility functions in such a way that the associated welfare function has a stationary point that corresponds to an inefficient equilibrium. In other words, for every prevention function, there is a set of parameter values for the utility function where the non-convexity occurs, and its complement for where it does not, and it is an empirical matter which case applies. Nonetheless, we conclude that, as the non-convexity can be generated relatively easily, it may not be neglected.



Next, we formulate conditions to ensure that the equilibrium solution of (2.3a-c) is efficient. As long as there is only one consumer, efficiency coincides with welfare optimality. We have seen in Proposition 1 that the equilibrium defines a stationary point of (2.1). It remains to show that it is unique and therefore a global optimum. As mentioned earlier, we consider two types of approaches that eliminate the non-convexity, one that restricts the functional form of utility and prevention functions, the other that introduces the institution of standard-based labeling as a filter on possible distributions, essentially discretizing the prevention technology.

3.2 Restrictions on functional forms

In this section we use the following property of concave functions:

Lemma 1 (extended function): (i) Assume that the function $f : R_+^n \to R_+$ is continuous, concave on R_+^n and the extended function $\tilde{f}(x,y) = yf(x/y)$, is continuous at $y = 0$ for any given nonnegative *x*, then $\tilde{f}$ is concave and homogeneous of degree one; (ii) if f is also non-stationary and non-decreasing, this property carries over to $\tilde{f}$ *;* finally, (iii) if f is also strictly concave, then $\tilde{f}$ is stricly quasiconcave (Ginsburgh and Keyzer, 1997, Theorem A.1.5).◊

Next, we reduce program (2.1) to an unconstrained form. Through Assumption U1, state 1 becomes strictly superior whenever consumption is positive. Hence, all probability constraints hold with equality and we obtain the form:

$$W^* = max_{e,x\geq 0} \sum_s \hat{P}_s u_s(x) + \sum_{s\neq 1} G_s(e) f_s(x) - (p^c + e)x \,. \qquad (3.4)$$

Clearly, the non-convexity disappears if we explicitly assume that $\sum_{s\neq 1} G_s(d/x) f_s(x)$ is concave, for $e = d/x$. Alternatively, we could modify the specification by supposing that prevention is expressed per unit of utility increase f_s, rather than of consumption *x*, that is as $G_s(d/f_s)$,

$$W(x,d) = (1 - \sum_{s\neq 1} \hat{P}_s) u_1(x) + \sum_{s\neq 1} \hat{P}_s u_s(x) + \sum_{s\neq 1} G_s(d/f_s(x)) f_s(x) - p^c x - d \,, \qquad (3.5)$$

and impose:

Assumption P2 (extended prevention): For every *s*, $\tilde{G}_s(x,d) = xG_s(d/x)$ is continuous at $x = 0$, for all nonnegative *d*.◊

Now, by Lemma 1 and Assumption P2, $\tilde{g}_s(d, f_s) = G_s(d/f_s) f_s$ is concave and hence, for concave increasing f_s i, the composition $\tilde{\tilde{g}}_s(d,x) = \tilde{g}_s(d, f_s(x))$ is concave as well. However, this formulation has the disadvantage that it treats utility gain as a physical quantity. Alternatively, we may consider a restriction on the functional form of the average utility gain:

Assumption U3 (fixed average gain): The average utility gain $\psi_s(x) = (u_1(x) - u_s(x))/x$ is fixed. ◊

It is now easy to prove:

Proposition 2 (uniqueness and efficiency): *Let assumptions P1, U1, U2 and U3 hold. Then, equilibrium model (2.3) has a unique equilibrium. This equilibrium is efficient.*

Proof. Existence of equilibrium was shown in Proposition 1. Since the utility gain and prevention functions are non-decreasing, we may write them as inequality constraint, and (2.1) becomes, using (2.4)-(2.6):

(a) $\quad max_{d,x\geq 0} \sum_s \hat{P}_s u_s(x) + \sum_{s\neq 1} \psi_s(x) g_s(d,x) - p^c x - d \,,$

for $g_s(d,x) = xG_s(d/x)$, which is concave, by Assumption P1 Lemma 1 (here we do not need assumption P2, because x is positive). Since $\psi_s(x)$ is fixed by Assumption U3, it follows that the objective of (a) is strictly concave. Therefore, it has a single stationary point, which corresponds to a global optimum. □

We note that for fixed average gain $\tilde{u}'(x) = 0$, the quadratic term in (3.3) vanishes. Conversely, if the gain is not fixed, the term persists, suggesting that it may be difficult to weaken the requirements of this proposition. Yet, we can also prove concavity in the special case where both the utility difference functions and the prevention function are of Cobb Douglas form:

$$G_s(d/x) = min[a_s(d/x)^{\alpha_s}, \eta_s] \text{ and } f_s(x) = b_s x^{\beta_s}, \tag{3.6}$$

with positive coefficients, $\eta_s < \hat{P}_s, \alpha_s \leq 1$, and $1 \leq \alpha_s + \beta_s < 2$. Welfare is now:

$$W(x,d) = (1 - \sum_{s \neq 1} \hat{P}_s) u_1(x) + \sum_{s \neq 1} \left(\hat{P}_s u_s(x) + min[a_s(d/x)^{\alpha_s}, \eta_s] b_s x^{\beta_s} \right) - p^c x - d \tag{3.7}$$

At given (x,d), the term for $s = 1$ is concave in x; for states $s \neq 1$ such that the bounds η_s are non-binding, the prevention term becomes concave in (x,d) and since $u_s(x)$ is also concave, the sum is concave. For states where the bound is binding, probability is fixed and the concavity of $u_s(x)$ ensures concavity. Hence, $W(x,d)$ is concave.

3.2 Standard-based labels

Eliminating non-convexity through restrictions on functional forms would seem less satisfactory, because it modifies the model without empirical justification. In contrast, the introduction of standard-based labels represents a change in the institutional setting of the economy itself. It avoids interaction effects discretizing the space of probabilities into a finite list of certified varieties. To highlight the influence of this discretization, we neglect the reduction in uncertainty, considered in Jones and Hudson (1996), that is due to the labeling itself even though this might well be the primary aim. In other words, we choose points that lie on the continuous probability distribution, with associated input costs. Any such point is likely to have a smaller variance than the original distribution, since it lacks the tails of the distribution, but we abstract from the possibility that labeling itself is likely to shift make the distribution and make it more favorable.

As mentioned above, it is not unusual for discretization to eliminate non-convexities. For example, if in Figure 1, attention was limited to R discrete choices with consumption $\hat{x}_r$, it would be straightforward to avoid the local minimum and find an approximate global optimum by choosing $r^* = arg\, max_r W(\hat{x}_r)$. However, this would involve central decision making, as the consumption choice would have to be made by a social planner. Labeling is a form of discretization that permits to achieve the same result in a more decentralized way, essentially because it allows standards to co-exist.

Suppose that the prevention technology distinguishes R techniques or regimes indexed r, with fixed prevention requirements $\hat{e}_r$ per unit of demand and with fixed probabilities $\hat{P}_{rs}$, $\sum_{s\neq 1}\hat{P}_{rs} < 1$. The retailer in (2.3b) now chooses the optimal label:

$$
\begin{aligned}
& max_{P_s \geq 0, all\ s; w_r \geq 0, all\ r; e\geq 0} \sum_s p_s P_s - (p^c + e) \\
& subject\ \ to \\
& P_s \geq \sum_r \hat{P}_{rs} w_r\,,\ s \neq 1 \\
& \sum_s P_s = 1 \\
& \sum_r w_r = 1 \\
& e = \sum_r w_r \hat{e}_r\,.
\end{aligned}
\tag{3.8}
$$

Any specialized solution (a solution with weight w_r equal to zero or unity) follows the underlying prevention function of Assumption P1. And since the function G_s is concave, even a non-specialized solution will approximate it arbitrarily closely, for a fine enough grid of labels. Hence, the discretization is flexible, although it limits the options for the retailer, and will never yield a solution superior to the global optimum of the original, non-discretized welfare program. We can state and prove:

Proposition 3 (discrete standards): *Consider equilibrium model (2.3a), (3.8), (2.3c) and let assumptions U1 and U2 hold. Then, (i) this model has an equilibrium and (ii), for almost all parameter values* $\hat{P}_{rs}$*, this equilibrium is efficient.*

Proof.

(i) Existence. Assumption U2, ensures positive consumption and boundedness, as in Proposition 2.We substitute the constraints of (3.8) into (2.1):

$$max_{P_s \geq 0, all\ s; w_r \geq 0, all\ r; e, x \geq 0} \sum_s P_s u_s(x) - (p^c + e)x$$

$$subject\ \ to$$

(a) $$P_s \geq \sum_r \hat{P}_{rs} w_r\ ,\ s \neq 1$$
$$\sum_s P_s = 1$$
$$\sum_r w_r = 1$$
$$e = \sum_r w_r \hat{e}_r\ .$$

This program has a global optimum, that, by the same reasoning as in the proof of Proposition 1, defines an equilibrium.

(ii) Specialization and efficiency. Next, after defining $\hat{P}_{r1} = 1 - \sum_{s \neq 1} \hat{P}_{rs}$, we formulate a program that for almost all parameter values $\hat{P}_{rs}$ yields the same optimum as (a) but differs in that it distinguishes consumption by regime:

(b1) $$V(\sigma_1^*,....,\sigma_R^*) = max_{w_r \geq 0, all\ r}\{\sum_r w_r \sigma_r^* \mid \sum_r w_r = 1\},$$

for given surplus

(b2) $$\sigma_r^*(\hat{P}_r) = \ max_{x_r \geq 0} \sum_s \hat{P}_{rs} u_s(x_r) - (p^c + \hat{e}_r)x_r\ ,$$

where $\hat{P}_r$ denotes the vector $(\hat{P}_{r1},...,\hat{P}_{rS})$. Since (b1) is a linear program, its solution set is convex. The optimal $(w_1,...,w_R)$ is the sub-gradient of the convex function V (Envelope theorem) and is single-valued almost everywhere on the set of possible utilities (Rademacher Theorem, see Clarke et al., 1998). To prove that the specialization property holds on the product $\prod_{r=1}^{R} \wp$ of simplices $\wp = \{(\mu_1,...,\mu_S) \mid \mu_s \geq 0, \sum_s \mu_s = 1\}$ of probabilities, we restrict probabilities to $\hat{P}_{r1}(\phi_r) = \phi_r$ and $\hat{P}_{rs}(\phi_r) = (1-\phi_r)\overline{P}_{rs}$ for $s \neq 1$, (where $\overline{P}_{rs}$ are fixed probabilities conditional on $\hat{P}_{r1}$ and with $\sum_{s \neq 1} \overline{P}_{rs} = 1$), and we define, the *R*-to-*R* vector function $\Phi = \prod_{r=1}^{R}[0,1] \to R_+^R\ ,\ Z(\phi) = (\sigma_1^*(\hat{P}_1(\phi)),...,\sigma_R^*(\hat{P}_R(\phi)))$, for fixed x_r. By strict concavity of utility, x_r is unique in (b2), hence this fixed value is unique. Since, by assumption U1, the safe state is superior, and in equilibrium all consumer goods are in positive demand, σ_r^* is increasing in ϕ_r and, therefore, invertible with respect to it, proving that the almost everywhere property holds on Φ. As this holds under the stated restriction on probability, it necessarily holds on the product of simplices. Furthermore, at a specialized optimum, the solution *(x, d)* in (b1-b2) coincides with that of (a). Consequently, at a specialized optimum, the stationary points of (a) support the equilibrium (2.3a), (3.8), (2.3c). Finally, since (b1) is a convex program, any stationary point is a global optimum, and efficiency follows. □

We note that the original welfare program (2.1) can be approached as a *maximax* or nested optimization problem. In (3.1) we solve for prevention as the inner problem and this leads to the non-concavity of $W(x)$ in the maximization over x in the outer problem. The common way of dealing with such non-convexities is to discretize the outer maximization, which is precisely what is done in this proof, where surplus maximization (b1) solves the inner optimization, while linear program (b2) deals with the discrete choice on prevention.

In addition, program (b1) shows that labeling avoids the need to extract the Lindahl price of characteristics from the consumer, as it eliminates nonrivalry across states. However, when in section 3 we extend the model to include several consumers, the non-rivalry will resurface, across consumers. Labeling also takes care of the prevention decision. Indeed, program (b1)-(b2) could be interpreted as a pure consumer problem, in which the agent opts, without any need for a retailer, for the product with the label that suits him best, buying at price $p_r^f = p^c + \hat{e}_r$.

Finally, there is no strong reason for insisting on specialization, as it seems natural to allow for mixed strategies, whereby the consumer buys several labels at the same time and where the mix of labels itself can be looked at as a consumer good. In fact, it is possible tot reinterpret program (b1)-(b2) as a procedure to find a maximal consumer surplus $\sigma = U(X_1,...,X_R,w_1,...,w_R) - \sum_r p_r^f X_r - \phi \sum_r w_r$, where ϕ is the multiplier on the constraint $\sum_r w_r = 1$ and the utility function is defined as

$$U(w_1,...,w_R,X_1,...,X_R) = \sum_r U_r(w_r,X_r) \tag{3.9a}$$

with

$$U_r(w_r,X_r) = w_r \sum_s \hat{P}_{rs} u_s(X_r / w_r). \tag{3.9b}$$

By Lemma 1 this utility function is concave, provided utility functions satisfy a continuity requirement as in P2. In other words, in the absence of labeling, the consumer can only buy one product, with a fixed probability distribution. Every purchase is a random drawing from the same probability distribution and hence with the same expected utility. Labeling enables the consumer to buy products with different probability profiles, and through it to improve his utility, not because of his love of variety in the sense of Dixit and Stiglitz (1977), but because this allows him to reach the convex hull of an action space that would be non-convex otherwise.

3.3 Modifications

In this section, we consider two modifications of the model itself that also rule out non-convexity. Besides their relevance in their own right, these modifications may clarify the nature of the non-convexity.

Delayed purchase. The non-rivalry disappears if the consumer facing uncertainty is able to delay deliveries until the state of nature has been revealed. In this case, he is freed from any holdup problem, as he can buy a different quantity of the product in every state, purchasing a quantity x_s at the observed price p_s. Since prevention necessarily takes place before purchase, it has to be independent of consumption, i.e. obey a function of total inputs *d*, rather than *e*, taken to be strictly concave. Hence, the program becomes:

$$\begin{aligned} & max_{P_s \geq 0, all\ s; d \geq 0} \sum_s P_s \sigma_s^* - d \\ & \quad P_s \geq \hat{P}_s - G_s(d), s \neq 1 \\ & \quad \sum_s P_s = 1, \end{aligned} \tag{3.10}$$

for given surplus $\sigma_s^* = max_{x_s \geq 0}(u_s(x_s) - p_s x_s)$, and this program is obviously convex. The non-convexity is also eliminated if scale independence is replaced by negative scale dependence, whereby $G_s(x,d)$ is concave non-decreasing, reflecting that higher scale *x* increases the productivity of prevention inputs *d* instead of lowering it.

Demand for sure characteristics. Rather than referring to an uncertain state, the index *s* could denote a physical product characteristic, say sweetness, that directly enters a concave utility function as $u(X_1,...,X_S)$. We now face a problem of optimal product design and program (2.1) reduces to the straight surplus maximization:

$$\begin{aligned} & max_{e,y \geq 0, X_s, Y_s \geq 0, all\ s}\ u(X_1,...,X_S) - (p^c + e)y \\ & subject\ \ to \\ & \quad X_s = Y_s, \qquad\qquad (p_s) \qquad . \\ & \quad \sum_s Y_s \leq y \qquad\qquad (p^f) \\ & \quad Y_s \geq \overline{P}_s(e)y, \qquad s \neq 1. \end{aligned} \tag{3.11}$$

If we substitute d/y for *e*, this program becomes convex. The retailer will be engaged in regular production of characteristics but, in view of the assumed constant returns to scale, not make any profit. Thus, it would seem that the optimal composition of the quality mix does not pose any problem of market failure. However, restrictions on the possibility to supply tailor made goods to every consumer separately will in the next section be seen to cause the non-convexity to re-emerge. When discussing the various options in the sequel, we also check whether the proposed solutions apply to modifications (3.10)-(3.12).

Constant returns in prevention. Suppose that the prevention function requires state-specific inputs e_s in (2.3b) and has constant returns to scale (meaning linear as long as e_s is a scalar):

$$\begin{aligned}
&max_{P_s \geq 0, all\ s; e \geq 0} \textstyle\sum_s P_s p_s - (p^c + e) \\
&subject\ \ to \\
&P_s \geq \hat{P}_s - \kappa_s e,\ s \neq 1 \qquad\qquad (\psi_s) \\
&\textstyle\sum_s P_s = 1. \qquad\qquad\qquad\quad (\rho)
\end{aligned} \tag{3.12}$$

The program will, for almost all prices p_s either choose $e = 0$, $P_1 = 1 - \sum_{s \neq 1} \hat{P}_s$, $P_s = \hat{P}_s$ for $s \neq 1$, or $e = max_{s \neq 1} [\hat{P}_s / \kappa_s]$, $P_1 = 1$, and $P_s = 0$ for $s \neq 1$, depending on whether the inequality $1 > \sum_{s \neq 1} \psi_s \kappa_s = \sum_{s \neq 1} (p_1 - p_s) \kappa_s$ holds. Hence, although the nonconvexity is not eliminated, we have only two discrete alternatives to deal with, and can choose the one with the highest welfare in (2.1). This is the form implicit in many of the game theoretic applications, where playing it safe or unsafe becomes a binary strategy choice. However, since the issues in food safety are never that clearcut, we further discard this option.

Discrete consumer choice. Finally, a natural alternative to the previous modification is to suppose that the consumer faces a discrete choice. In particular, if the decision is whether to purchase or not, there is no non-convexity left once the purchase is know to take place. This approach is followed in much of the literature on product safety and quality choice (for an overview, see e.g. Deaton and Muellbauer, 1994, section 10.3). However, while this may seem a relevant perspective for the individual consumer, from the producer's angle there is still an interaction to address between the number of buyers, and the safety or quality of the product. Moreover, indivisibility of the consumer good is less relevant in relation to food safety. Hence, we discard this option as well.

Section 4
Partial equilibrium with production

So far, the product was taken to be available to the retailer at a given price raw material price p^c. However, it appeared in section 2 that in an efficient equilibrium, the profit from prevention (2.4) does not reflect in the price margin between consumer and raw material price. Hence, an independent retailer purchasing at given raw material price would not be able to make a profit in prevention, unless he could find a way of recovering it from the raw material suppliers. This will not be difficult if these cannot bypass him and have to conform to his product specifications, but in this case competition itself will be threatened, as prevention and production become complements. In this section, we make this aspect explicit by describing the production process of the raw material itself and its relation to prevention. We initially treat production as a single process, but later on we distinguish various stages, to highlight that through the prevention-related activities, further complementarities arise along the processing chain that foster vertical integration.

We introduce a profit maximizing producer with a concave, homogeneous, and continuous production function $f : R_+ \to R_+$, solving

$$max_{q,v\geq 0}\{ p^c q - v \mid q \leq f(v)\}. \tag{4.1}$$

and a market equilibrium condition for the consumed commodity:

$$y \leq q \tag{4.2}$$

with clearing price p^c for the raw material before prevention. The partial equilibrium model (2.3) is now extended with (4.1) and (4.2). If we bypass Lindahl pricing, the associated welfare program (2.1) is modified to:

$$\begin{array}{lll} & max_{P_s\geq 0, all\ s;\ d,q,v,x,y\geq 0} \sum_s P_s u_s(x) - d - v & \\ subject\ to & & \\ & x \leq y & (p^f) \\ & P_s y \geq \hat{P}_s y - G_s(d/y)y,\ s \neq 1 & (\psi_s) \\ & \sum_s P_s = 1 & \\ & y \leq q & (p^c) \\ & q \leq f(v), & \end{array} \tag{4.3a}$$

where the probability constraints are multiplied by y to obtain the balance of characteristics. It may be verified that price relation (2.3c) holds as before. To derive the decentralization of Proposition 1, we rewrite (4.3a), for optimal shadow prices p^f and p^c, and with e instead of d, as:

$$\begin{aligned} & max_{e,x\geq 0,P_s\geq 0,\, all\ s}\, [\textstyle\sum_s P_s u_s(x) - p^f x] + [p^f y - (p^c + e)y] + [p^c q - v] \\ & subject\ \ to \\ & P_s y \geq \overline{P}_s(e)y \qquad\qquad (4.3b) \\ & \textstyle\sum_s P_s = 1 \\ & q \leq f(v), \end{aligned}$$

where the second term in square brackets is zero. This welfare program maximizes the sum of consumer and producer surplus, where the producer price p^c is lowered by the margin e, relative to p^f that acts like a transportation cost, but excludes the profit (2.4) on prevention, as in (2.3c), that accrues to the supplier of the raw material. For every case of the previous section, program (4.3a) will have a unique stationary point, and existence and efficiency of equilibrium follow.

Multiple producers

While maintaining a formulation with a single consumer and a single consumption good, we consider J firms, indexed j. Two possibilities should be distinguished. If the retail sector exercises a single prevention policy, or if the uncertainty itself affects all output uniformly, it suffices to define a production function by firm , $f_j(v_j)$, with associated profit maximization and to insert commodity balance $y \leq \sum_j q_j$. Consumers and producers will decide on the basis of commodity price p^f and p^c, respectively. Only the retailer will need the prices of characteristics, and with the labeling of Proposition 3, these can be dispensed of as well. Efficiency will be ensured under the conditions of propositions 2 and 3, and models (3.7), (3.10)-(3.11). Furthermore, by distinguishing several consumer goods, it would be possible to represent different profiles, as will be seen in the next section.

Alternatively, one may suppose that producers deliver characteristics that can be blended, because only the average content affects the consumer. In this case, every firm can supply mixes of safety characteristics in different proportions P_{js}, while individually taking care of prevention and possibly imposing standards. Now , dropping the distinction between q and y for notational convenience, and supposing that individual firms have access to the prices of characteristics p_s, that are inclusive of raw material cost, we write for firm j:

$$\Pi_j(p_1,...,p_S) = max_{P_{js}\geq 0, all\ s;\ d_j,v_j,y_j\geq 0} \sum_s p_s P_{js} y_j - d_j - v_j$$

$$subject\ \ to$$

$$P_{js} \geq \hat{P}_{js} - G_{js}(d_j/y_j), s \neq 1 \quad (4.4)$$

$$y_j \leq f_j(v_j)$$

$$\sum_s P_{js} = 1.$$

This can be rewritten in terms of the probability weighted production $Y_{js} = P_{js} y_j$ as:

$$\Pi_j(p_1,...,p_S) = max_{Y_{js}\geq 0, all\ s;\ d_j,v_j,y_j\geq 0} \sum_s p_s Y_{js} - d_j - v_j$$

$$subject\ \ to$$

$$Y_{js} \geq \hat{P}_{js} y_j - G_{js}(d_j/y_j) y_j, s \neq 1 \quad (4.5)$$

$$y_j \leq f_j(v_j)$$

$$\sum_s Y_{js} \leq y_j,$$

or in more general and compact form, with the single constraint:

$$Y_{js} \geq F_{js}(d_j, v_j). \quad (4.6)$$

We note that whereas the production function f_j could be replaced by a transformation function, this is not possible for prevention, since a standard (quasiconvex, nonincreasing) transformation constraint $F_j(-Y_{j1},...,-Y_{jS}, d_j, v_j) \leq 0$, would allow for free disposal of specific outputs, which is not acceptable for safety characteristics. The corresponding welfare program reads:

$$max_{P_s, Y_{js}\geq 0, all\ s;\ d_j, v_j \geq 0, all\ j; x\geq 0} \sum_s P_s u_s(x) - \sum_j d_j - \sum_j v_j$$

$$subject\ \ to$$

$$P_s x \geq \sum_j Y_{js} \qquad (p_s) \quad (4.7)$$

$$\sum_s P_s = 1$$

$$Y_{js} \geq F_{js}(d_j, v_j),$$

and defines the price of characteristics p_s for all *s*.

Multiple firms and blending of characteristics: sufficient conditions for efficiency

We check whether the cases of the previous section carry through. Because prevention is firm-specific, expression (3.4) no longer applies, and the Cobb Douglas form (3.7) and the form (3.10) with scale independent prevention cannot be used. Uniqueness of the stationary point is now to be established for (4.7) directly.

First, the case with fixed average gain of Proposition 2 is dealt with by substituting $X_s = P_s x$ and rewriting expected utility as $\sum_s P_s u_s(x) = u_1(x) + \sum_s X_s \tilde{p}_s$ for the expected utility in the objective.

Secondly, the labeling of Proposition 3 can readily accommodate production with blending of characteristics. However, since prevention now also appears as part of the regular, divisible technology, we may suppose that the cost of labeling is already covered by producer, which amounts to requiring that they are independent of the regime, and we can write the convex welfare program:

$$
\begin{aligned}
& max_{w_r, X_r \geq 0, all\ r;\ d_j, v_j, Y_{js} \geq 0, all\ j,s} \sum_s w_r \hat{P}_{rs} u_s(X_r / w_r) - \sum_j d_j - \sum_j v_j \\
& subject\ \ to \\
& \quad \sum_r \hat{P}_{rs} X_r = \sum_j Y_{js} \qquad\qquad (p_s) \qquad\qquad (4.8) \\
& \quad \sum_r w_r = 1 \\
& \quad Y_{js} \geq F_{js}(d_j, v_j),
\end{aligned}
$$

where $X_r = w_r x_r$ is the weighted demand. By Lemma 1, the objective is concave provided it meets a continuity requirement as in P2, and like in Proposition 3, the specification will for almost all values of probabilities in the simplex specialize on a single regime *r* and yield a global optimum of the welfare program with endogenous choice of label. In this economy with one commodity and one consumer there is no scope for pluralism, as all producers have to comply to the same standard.

Finally, modification (3.11) with demand for characteristics could be represented in (4.7) if we replace $P_s x$ by X_s in the constraint of (4.7) and replace the expected utility by the utility from characteristics in the objective.

Contamination

Product safety rarely is the affair of a single production unit and more often affects a long production chain from the raw material producer to the consumer. As an intermediate case between the single standard imposed by the retailer in (4.3) and the tradable characteristics in (4.4), one could envisage production chains that internally trade in characteristics, while facing given standards externally. For instance, we may consider chain *c*, supplying a single output, say, tons of hog meat. The various physical inputs of firms *j*, can be represented as v_{cj} in the chain's production function $f_c(v_{c1},...,v_{cJ})$. With respect to the safety of the product, it effectively becomes a Markov chain, and the nature of the hazard will determine whether upstream events can be redressed downstream by means of additives or cleaning operations. However, this kind of substitution is generally difficult when health aspects or product reputation are at stake, say, in relation to environmental substitutability, animal friendliness or labor standards, where good performance in one segment cannot compensate for poor performance in another. This complementarity can be represented via the constraint:

$$P_{cs} \geq max_j(\hat{P}_{cjs} - G_{cjs}(d_{cj} / y_c)), \quad s \neq 1,$$

and amounts to a modification of profit maximization (4.4) to:

$$\Pi_c(p_1,...,p_S) = max_{P_{cs} \geq 0, all\ s;\ d_{cj}, v_{cj} \geq 0, all\ j; y_c \geq 0} \sum_s p_s P_{cs} y_c - \sum_c d_{cj} - \sum_c v_{cj}$$

$$subject\ \ to$$

$$P_{cs} \geq max_j(\hat{P}_{cjs} - G_{cjs}(d_{cj}/y_c)), s \neq 1 \tag{4.9}$$

$$y_c \leq f_c(v_{c1},...,v_{cJ})$$

$$\sum_s P_{cs} = 1.$$

This can, like in (4.5), be rewritten as a convex program, in terms of the probability weighted production $Y_{cs} = P_{cs} y_c$, confirming that as long as prices of characteristics are available, prevention can be viewed as a technological process, and that the producer side of the model can accommodate a wide range of specifications.

At the same time, the specification also reveals that decentralization to individual firms within the chain will be problematic, because the prices of characteristics become firm-specific, and exhibit the same non-rivalry with respect to total delivery y_c as in the relation between the consumer and the retailer in section 2. In other words, because of the contamination, the inputs into prevention of the different firms in the chain become non-substitutable. Hence, the valuation of the deliveries can only be effectuated if they are accompanied by tight inspections, to measure P_{cjs}, and combined with an appropriate adjustment process to reveal the prices faced by the various segments, illustrating the tendency towards vertical integration.

Section 5
Partial equilibrium with heterogeneous consumers and multiple commodities

In this section we allow for heterogeneity among commodities and consumers.

Multiple commodities

We consider *K* commodities, indexed *k*. In models with multiple commodities, the sign switches in the determinant of the Hessian $W''(x_1,...,x_K)$ in (3.1a) will reflect the possible multiplicity of stationary points. The functional forms are easily adapted to deal with multiple commodities. The form with fixed average gain of Proposition 2 generalizes to:

$$u_s(x_1,...,x_K) = u(x_1,...,x_K) + \sum_k b_{ks} x_k \; . \tag{5.1}$$

It cannot accommodate effects across commodities, and i.e. states should be commodity-specific and their probabilities independent across commodities: P_{ks}, $\sum_s P_{ks} = 1$. Option (3.8) with labeling applies fully when x_r is a vector, and can accommodate stochastic dependence across commodities.

Multiple consumers

Next, we distinguish *I* different consumers indexed *i* and a single commodity. If markets for characteristics were consumer specific, producers could deliver to every market separately and the welfare criterion of, say, (4.7) would become $\sum_s P_{is} u_{is}(x_i) - \sum_j d_j - \sum_j v_j$, with balances for characteristics and prices p_{is}. However, it seems more realistic to focus on the situation where producers have to select probability profiles (bundles of characteristics) for a commodity that will be used by consumers whose preferences differ. This introduces a complication in the form of the non-convex restriction:

$$\frac{X_{is}}{\sum_s X_{is}} = \frac{\sum_i X_{is}}{\sum_i \sum_s X_{is}}, \text{ all } i,s \tag{5.2}$$

which is not readily eliminated, and obviously reduces welfare as compared to the situation of the Lancaster model (1966), where every consumer can buy exactly the preferred bundle of characteristics. In terms of the earlier welfare function (3.1)-(3.3), the non-convexity is reflected as follows. For *c* denoting the vector of consumption, welfare is:

$$W(c) = [V(\tilde{u}_1(c),...,\tilde{u}_S(c)) - p^C]c^T \iota \; , \tag{5.3}$$

and $\tilde{u}_s(c) = \sum_i u_{is}(c_i) / \sum_i c_i$, while c and ι are I-dimensional vectors. Hence, the welfare function has first derivative:

$$W' = x \sum_s V_s' \tilde{u}_s' + V\iota - p^c \iota , \tag{5.4}$$

where $x = \sum_i c_i$, c, p^c and V are scalars, V' is an S-dimensional vector, and $\tilde{u}_s' = (u_s' - \tilde{u}_s \iota) / x$ is an I-dimensional vector. The second derivative becomes

$$W'' = x(\tilde{u}'^T V'' \tilde{u}') + (\sum_s V_s' (\iota \tilde{u}_s'^T + \tilde{u}_s' \iota^T + x\tilde{u}_s'') , \tag{5.5a}$$

where $\tilde{u}_s''$ is the $I \times I$ Hessian of $\tilde{u}_s$ and $\tilde{u}'$ is an $S \times I$-dimensional matrix. Since $\iota\tilde{u}_s'^T + \tilde{u}_s' \iota^T + x\tilde{u}_s'' = u_s''$, this reduces to

$$W'' = x(\tilde{u}'^T V'' \tilde{u}') + (\sum_s V_s' u_s''), \tag{5.5b}$$

indicating, that, just like in (3.3), the non-convexity disappears if V is piecewise linear, with $V'' = 0$, and if $\tilde{u}' = 0$. The latter condition is more demanding than with a single consumer, because it requires that all consumers should have the same fixed average utility. Since, for a quadratic utility function with positive intercept as in Example 2, the first term can be made arbitrarily large, we can, as before, construct cases where the first term dominates the second. Concavity is lost once W'' has at least one positive eigenvalue[3] but whether this actually causes multiplicity of stationary points depends on further model properties.

Restrictions on the functional form

Turning to the restrictions where convexity is maintained, the case with a fixed average gain applies if the gain does not vary among consumers, since we can write

$$u_{is}(x_i) = u_i(x_i) + b_s x_i , \tag{5.6a}$$

which defines the aggregate utility criterion

$$U(x) = max_{x_i \geq 0, all\, i} \{ \sum_i u_i(x_i) \mid \sum_i x_i = x \} , \tag{5.6b}$$

[3] If we write A for the first term in (5.5b) and $-B$ for the second term, then, as A is positive semidefinite and B positive definite, both have nonnegative eigenvalues. A sufficient condition for non-concavity is that the smallest eigenvalue of A exceeds the largest eigenvalue of B.

and permits to use $U_s(x) = U(x) + b_s x$, as before. This formulation shows, like in Proposition 2, that only the average utility gain needs to remain fixed, as opposed to the utility level. It also indicates that differentiation of b_{is} across consumers would be impossible, since this would lead to an expression $\sum_s P_s U_s(x_1,...,x_I) = U(x) + \sum_s \sum_i P_s b_{is} x_i$, for expected utility that converts to:

$$\sum_s P_s U_s(x_1,...,x_I) = U(x) + \sum_s \frac{\xi_s}{x} \sum_i b_{is} X_{is}\,, \tag{5.6c}$$

for $x = \sum_i \sum_s X_{is}$ and $\xi_s = \sum_i X_{is}$ but is not concave in $(X_{11},...,X_{IS})$. Hence, we are left with labeling as only remaining option. As in program (b1) in the proof of Proposition 3, we can define the regime-specific surplus, now by consumer, $\sigma_{ir}^* = max_{x_{ir} \geq 0} \sum_s \hat{P}_{rs} u_{is}(x_{ir}) - p_r^f x_{ir}$, and $\sigma_r^* = \sum_i \sigma_{ir}^*$, for $p_r^f = p^c + \hat{e}_r$ for the inner maximization, and approximate the discrete outer maximization by the linear program:

$$W^* = max_{w_r \geq 0,\, all\ r} \{ \sum_r w_r \sigma_r^* \mid \sum_r w_r = 1 \}. \tag{5.7}$$

However, as announced earlier, this makes non-rivalry resurface, since the consumer cannot determine the choice of label w_r individually, and the producer will need to extract the Lindahl price ϕ_{ir} from the consumer problem:

$$\begin{aligned} & max_{w_{ir}, X_{ir} \geq 0} \sum_r w_{ir} \sum_s \hat{P}_{rs} u_{is}(X_{ir} / w_{ir}) - \sum_r p_r^f X_{ir} \\ & subject\ to \\ & w_{ir} = w_r\,, \qquad\qquad (\phi_{ir}) \end{aligned} \tag{5.8}$$

while the producer solves (5.7), since $\phi_{ir} = \sigma_{ir}^*$ and, hence:

$$\sigma_r^* = \sum_i \phi_{ir}\,. \tag{5.9}$$

A variant that avoids the non-rivalry, would drop the restriction in (5.8) and allow for w_{ir} to be chosen freely. As this generally improves welfare, it illustrates that the harmonization of labels tends to be detrimental. It may also be mentioned that in this model with heterogeneous consumers, the welfare optimum is Pareto-efficient: any possibility to improve the expected utility of one consumer without reducing that of another would contradict optimality.

Models with demand for sure characteristics

To represent demand for sure characteristics by heterogeneous consumers, we return to model (3.11):

$$
\begin{aligned}
& max_{e,y\geq 0,x_i,X_{is},P_s\geq 0,all\ i,s} \sum_i u_i(X_{i1},...,X_{iS}) - (p^c + e)y \\
& subject\ \ to \\
& \quad \sum_s X_{is} = x_i \\
& \quad X_{is} = P_s x_i, \\
& \quad \sum_i x_i \leq y \\
& \quad P_s \geq \overline{P}_s(e), \qquad s \neq 1 \\
& \quad \sum_s P_s = 1
\end{aligned}
\tag{5.10}
$$

It appears that this model also loses its convexity, because the individual demand has to meet the restrictions (5.2). Nonetheless, two options would seem to remain open. One is to develop the form (3.11) with scale independent prevention, while assuming, in addition, that the individual utility functions separate into

$$
u_i(X_{i1},...,X_{iS}) = Q(X_{i1}/x_i,...,X_{iS}/x_i)h_i(x_i), \tag{5.11}
$$

for aggregate quantity $x_i = \sum_s X_{is}$, where Q is nonnegative concave, and h_i strictly concave positive. The inputs in product design are now non-rival with respect to individual consumption. The formulation is restrictive because it supposes that all consumers value quality characteristics in the same way and may only differ in their quantity valuation, but it permits to write program (5.10) as:

$$
\begin{aligned}
& max_{d,x_i\geq 0,P_s\geq 0,all\ i,s}\ Q(P_1,...,P_S)\sum_i h_i(x_i) - \sum_i p^c x_i - d \\
& subject\ \ to \\
& \quad P_s \geq \underline{P}_s(d) \\
& \quad \sum_s P_s = 1,
\end{aligned}
\tag{5.12}
$$

which has, a pseudoconcave first term in the objective (a bi-nonlinear function, Avriel, p.156). However, even this property is insufficient to ensure pseudo-concavity of the objective. We remark, moreover, that in a multicommodity model, this property would be lost, unless all consumers share a common homothetic utility function. We formulate it, nonetheless, because, as will be seen in the next section, pseudo-concavity is maintained if consumers have to meet a budget constraint.

The other option is to apply labeling for determination of the optimal product design, where the parameters $\hat{P}_{rs}$ denote the mix of characteristics of design *r*, leading to the consumer decision:

$$max_{w_{ir},X_{ir}\geq 0} \sum_r w_{ir} u_i(\hat{P}_{r1} X_{ir} / w_{ir},...,\hat{P}_{rS} X_{ir} / w_{ir}) - \sum_r p_r^f X_{ir}$$

$$\text{subject to} \qquad (5.13)$$

$$w_{ir} = w_r, \qquad (\phi_{ir})$$

and with choice of label (5.9), we can as before distinguish situations with imposed standardization and free choice of label.

Section 6
General equilibrium

The transition from partial to general equilibrium is effectuated when, in consumer problem (2.3a), the expected surplus maximization is replaced by expected utility maximization subject to a budget constraint. This creates special problems in a model with endogenous product safety and composition, because it can have no free disposal of surpluses and, consequently, the price of characteristics may not be positive. It is not possible to eliminate excess fat or a high risk of salmonella at zero cost from a piece of chicken. Therefore, non-negativity of income is not assured, and standard proofs of existence and efficiency of general equilibrium do not apply. This problem arises independently of and in addition to the non-convexity. Our main aim in this section is to show that standard-based labeling can eliminate it under mild assumptions, by "packaging" negative prices with positive ones, under the fixed probability profile.

For ease of notation, we maintain commodity specific utility functions, i.e. suppose that expected utility is additively separable over commodities. Another typical feature is that when the price of a characteristic could be negative, consumers will not always want to market all the endowments they own. Therefore, we must allow for an endogenous sales fraction g_{ik} in the consumer model:

$$\begin{aligned} & max_{x_{ik}\geq 0,\; g_{ik}\geq 0, P_{ks}\geq 0, all\; k,s} \sum_k \sum_s P_{iks} u_{iks}(x_{ik}) \\ & subject\;\; to \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (6.1) \\ & \sum_k (\sum_s P_{ks} p_{ks}) x_{ik} \leq \sum_k g_{ik} p_k^o \omega_{ik} + \sum_j \theta_{ij} \Pi_j \qquad (\lambda_i) \\ & g_{ik} \leq 1 \\ & P_{iks} = P_{ks} \qquad\qquad (\sigma_{iks}) \end{aligned}$$

where ω_{ik} is the initial stock of commodity *k* and κ_{ks} the content property *s* of commodity *k*, and θ_{ij} is the fixed profit share of consumer *i* from firm *j*.

On the producer side, we account for the fact that firms may be confronted with different qualities of inputs, but we initially assume that they are, unlike consumers, in a position to perform the grading appropriately, and therefore know what to buy. They decide on purchases V_{hsjk} of inputs commodity *h* of quality *s*, to be used in the production of commodity *k* by firm *j*. Their profit is determined from:

$$\Pi_j = \max_{Q_{jks}, V_{hsjk} \geq 0, Y_{jks} \text{ all } k,s} \sum_k \sum_s p_{ks} Y_{jks}$$
$$\text{subject to} \qquad\qquad , \qquad (6.2)$$
$$Y_{jks} = Q_{jks} - \sum_h V_{hsjk}$$
$$Q_{jks} \geq F_{jks}(V_{11jk}, \ldots, V_{KSjk})$$

where Y_{jks} denotes the probability weighted net supply, and is negative for inputs. In the explicit form (4.5) with prevention functions, this becomes:

$$\Pi_j = \max_{Y_{jks} \text{ all } k,s} \{ \sum_k \sum_s p_{ks} Y_{jks} \mid (Y_{j11}, \ldots, Y_{jKS}) \in H_j \}, \qquad (6.3)$$

for

$$H_j = \{ (Y_{j11}, \ldots, Y_{jSK}) \mid Y_{jks} = Q_{jks} - \sum_h D_{hsjk} - \sum_h V_{hsjk}, \text{all } k,s;$$
$$q_{jk} \geq 0, \text{all } k; Q_{jks} \geq 0, Y_{jks} \text{ all } k,s; D_{hjsk}, V_{hjks} \geq 0, \text{all } h,k,s;$$
$$Q_{jks} \geq \hat{P}_{jks} q_{jk} - G_{jks}(D_{11jk}/q_{jk}, \ldots, D_{KSjk}/q_{jk}) q_{jk}, s \neq 1$$
$$q_{jk} \leq f_{jk}(V_{11jk}, \ldots, V_{KSjk}) \qquad (6.4)$$
$$\sum_s Q_{jks} \leq q_{jk} \}.$$

If the production and prevention functions f_{jk} and G_{jks} are concave and homogeneous, the production set H_j will be convex, and nonempty, with possibility of inaction ($0 \in H_j$). Its boundedness will be discussed below. Thus, this set has all basic properties from production theory. Yet, we do not require the underlying transformation function to be non-decreasing in all net supplies. This permits to represent that imperfect malleability of the quality mix via non-substitution in the production and prevention function, with poorer qualities possibly contributing negatively to output. Furthermore, the specification does not rule out that some firm might buy negatively priced inputs (polluted material) and transform it, with some other inputs, into a high-value product. The boundedness of the production set of the full economy, e.g. because of labor or time requirements, will keep this activity finite, and may eventually make the price of the input positive.

Hence, these consumer and producer models could be viewed as specific forms of the standard competitive equilibrium versions, but the major differences lie in the balances that link the various agents. First, there is no free disposal of net supply, and therefore, prices of characteristics may be negative, and, second, probability shares are endogenously determined jointly with these prices and in a non-rival way across consumers. Both properties can be represented simultaneously via the following expected surplus maximization, subject to the balances of characteristics:

$$max_{\tilde{P}_{ks}} \ \sum_s \tilde{P}_{ks} \sum_i \sigma^*_{iks}$$

subject to (6.5)

$$\tilde{P}_{ks} \sum_i x_{ik} = \sum_j Y_{jks} + \kappa_{ks} \sum_i g_{ik} \omega_{ik} \,, \qquad (\tilde{p}_{ks})$$

where only probabilities appear as decision variables, for, possibly negative, money metric consumer surpluses $\sigma^*_{iks} = \sigma_{iks} / \lambda_i$ communicated from (6.1). Note that normalization of prices on the simplex may be impossible, since for every commodity *k*, the first-order conditions imply that $\tilde{p}_{ks} = \sum_i \sigma^*_{iks}$, which could be negative for some *s*. Obviously, equilibrium conditions also require the aggregate balances to match and for this consumer prices p_k^f adjust until

$$\sum_i x_{ik} = \sum_j \sum_s Y_{jsk} + \sum_i g_{ik} \omega_{ik} \,, \tag{6.6}$$

while

$$p_{ks} = \rho_k \ \tilde{p}_{ks} \,, \tag{6.7a}$$

for scaling factor ρ_k such that

$$p_k^f = \sum_s P_{ks} \, p_{ks} \tag{6.7b}$$

and

$$p_k^o = \sum_s p_{ks} \kappa_{ks} \,, \tag{6.7c}$$

which ensures that, in equilibrium

$$P_{ks} = \tilde{P}_{ks} \text{ and} \tag{6.7d}$$
$$P_{ks} \geq 0, \sum_s P_{ks} = 1 \,. \tag{6.7e}$$

In short, consumers decide at given prices and probabilities, while the producers determine through their product mix, the probabilities faced by the consumer.

Negishi format

Since prices of characteristics might be negative, a proof of existence of equilibrium of the model (6.1)-(6.7) is problematic, essentially because incomes may become negative in the consumer problem, causing its infeasibility. The problem does not arise in the alternative, Negishi format that relies on a welfare program subject to physical constraints, with utilities aggregate into a welfare objective through weights α_i and adjusted on the simplex, until every consumer meets his budget. Hence, the budget constraints are only required to hold in equilibrium. The social planner maximizes:[4]

$$W = \sum_i \alpha_i \sum_k \sum_s P_{ks} u_{iks}(x_{ik}), \tag{6.8}$$

subject to (4.7), (6.4),(6.6) as well as the constraints of (6.6). This leads us back to the earlier problem of a non-concave objective and before returning to labeling, we check whether we can tackle these through restrictions on functional forms.

Restrictions on functional form

It appears that even a fixed average gain that is uniform across consumers cannot eliminate the non-convexity, since the welfare weight is necessarily consumer specific, as in this case the welfare function becomes

$$W = \sum_k \sum_i \alpha_i u_{ik}(x_{ik}) + \sum_k (\sum_s P_{ks} b_{ks})(\sum_i \alpha_i x_{ik}), \tag{6.9}$$

where the second term indicates that, for reasons similar to those in (5.6c), now with the heterogeneity caused by the welfare weights, it is not possible to write the expression as a concave form in (P_{ks}, X_{ks}). The problem does not arise if probabilities are allowed to differ across consumers, but we disregard this case.

Regarding demand for characteristics, it appears that if there is a single commodity, model (5.10) applies, with, unlike before in (5.10) a pseudo-concave welfare objective $W = Q(P_{11},...,P_{1S}) \sum_i \alpha_i h_{i1}(x_{i1})$. However, this specification is even more restrictive.

We conclude that while in model (6.1)-(6.8) excessively strong restrictions are needed to keep incomes positive out of equilibrium, the Negishi format cannot be used because the non-convexity undermines the continuity properties of the budget surplus functions.

Labeling

Since discrete labels emerge as the only remaining option, we elaborate on the specification of this case. We limit the discussion to the case with a commodity-specific standard. The extension to commodity and consumer specific standards is straightforward. The commodity-specific standards only apply to aggregate sales to consumers, but we will subsequently show that the

[4] Concavity of the additive welfare function is not a necessary condition for efficiency. It is sufficient if in $min_i [\alpha_i \sum_k \sum_s P_{ks} u_{iks}(x_{ik})]$ we can eliminate the product form and obtain a continuous, quasiconcave form.

model is readily adapted to represent the situation where chains only price characteristics for their internal trade and maximize profits at commodity prices p^c of standardized products.

In earlier sections, we have seen that in a partial equilibrium setting, labeling serves to discretize the outer optimization problem of the welfare program, written in maxi-max form. In a general equilibrium model, the situation becomes more complex, since the introduction of discrete choice tends to undermine existence of a competitive equilibrium, and to require public intervention. Clearly, the continuous weight variable w_{kr} permits to avoid the non-convexity, and this creation of divisibility restores efficiency. But in a model with endogenous prices, it may not end up in a specialized solution.

Although the option with discrete labels can accommodate a general utility function and stochastic dependence of hazards across commodities, and while noting that this is the relevant specification to implement the standards in relation to choice of characteristics in (5.11), we nonetheless, impose additive separability on the utility function over commodities and stochastic independence, since this keeps the presentation more transparent. The consumer problem for purchasing of goods with commodity specific labels is:

$$\begin{aligned} & max_{w_{ikr} \geq 0, X_{ikr} \geq 0, all\ kr; g_{ik} \geq 0, all\ k} \sum_k \sum_r \sum_s w_{ikr} \hat{P}_{krs} u_{iks}(X_{ikr} / w_{ikr}) \\ & subject\ \ to \\ & \sum_k \sum_r p^f_{kr} X_{ikr} \leq \sum_k g_{ik} p^o_k \omega_{ik} + \sum_j \theta_{ij} \Pi_j \qquad (\lambda_i) \\ & w_{ikr} = w_{kr} \qquad (\tilde{\phi}_{ikr}) \\ & g_{ik} \leq 1 \end{aligned} \quad , \tag{6.10}$$

are the consumer price of sold endowments and purchased goods, respectively. Producers behave in accordance with (6.3) but we must still include the retailer who like in (5.9) chooses the labels given the (Lindahl) price of characteristics $\phi_{ikr} = \tilde{\phi}_{ikr} / \lambda_i$ from (6.10):

$$\begin{aligned} & max_{w_{kr} \geq 0, all\ r} \sum_s w_{kr} \sum_i \phi_{ikr} \\ & subject\ \ to \\ & \sum_r w_{kr} = 1. \end{aligned} \tag{6.11}$$

This program shows that the label will only be selected if it yields the highest surplus, not on a per capita basis but in aggregate terms. The balance of characteristics becomes:

$$\sum_r \hat{P}_{krs} \sum_i X_{ikr} = \sum_j Y_{jks} + \kappa_{ks} \sum_i \omega_{ik} \tag{6.12}$$

and has clearing price p_{ks} associated to it, as well as aggregate prices:

$$p^f_{kr} = \sum_s p_{ks} \hat{P}_{krs} \text{ , and} \tag{6.13a}$$

$$p_k^o = \sum_s p_{ks} \kappa_{ks} \tag{6.13b}$$

Through summation over states, the commodity balance in quantity terms:

$$x_k = \sum_j y_{jk} + \sum_i \omega_{ik} \ , \tag{6.14}$$

where $x_k = \sum_r \sum_i X_{ikr}$ and $y_{jk} = \sum_s Y_{jks}$ .Next, we introduce assumptions to characterize the functions and the endowments of this model. We modify assumption U1 on utility as follows:

Assumption U4 (utility): (i) For every *i*, *k*, *s* the utility function $u_{iks} : R_+ \to R$ is continuous; (ii) for every *i*, *k*, *r*, there is a nonnegative (possibly infinite threshold) level $\hat{x}_{ikr}$ such that the expected utility functions $U_{ikr}(x_{ikr}) = \sum_s \hat{P}_{krs} u_{iks}(x_{ikr})$ is strictly concave increasing below this level, and non-increasing beyond it; expected utility also satisfies $lim_{y \downarrow 0} \ yU_{ikr}(x / y) = 0$ given any $x \geq 0$. (iii) for every *i*, *r*, there is a nonempty subset K_i^c of commodities for which expected utility is increasing at all consumption levels; (iv) for $i = 1$ expected utility is increasing in every *r* and $k \in K^c = \bigcup_{i=2}^{I} K_i^c$ and *r*.◊

This relaxes assumption U1, to the extent that the utility function could now be nonconcave in some unsafe states, provided this is compensated for on average (condition (ii)). This is desirable because the concavity assumption has, via the weak axiom, its basis in the revealed preference over commodity purchases, and not in preference relations themselves (Varian, 1992). Since excess consumption can always be harmful in many ways, we allow for non-concavity of the demand beyond the threshold, when the function is non-increasing. The threshold level could be zero for commodities that are always harmful to a particular consumer but we exclude oscillations in the derivative. The strict concavity is imposed because it simplifies leads to a single-valued demand and because strict concavity for one state already leads to strict concavity for the expected utility. Condition (iii) is an addition to U1. It ensures non-satiation of every consumer, so that everyone will fully spend his income. Finally, condition (iv) amounts to requiring that any regime should be banned which would even on average not lead to increasing utility for the possibly hypothetical and arbitrarily small consumer $i = 1$. Under this specification of utility, consumers may choose not to buy a particular commodity that they find too expensive, possibly because of its high labeling costs. We remark that model (5.8) with standards on characteristics also fits within this framework if we define the regime specific utility function: $U_{ikr}(x_{ikr}) = u_{ik}(\hat{P}_{kr1} x_{ikr} ,..., \hat{P}_{krS} x_{ikS})$. Extension to a multi-commodity form is readily dealt with as well.

We also introduce an assumption to ensure that all consumers have positive income when all prices are positive.

Assumption E1 (endowments): Consumers only own endowments ω_{ik} of commodities $k \in K^o$; (i) these commodities are not produced: $Y_{jks} \leq 0$ for any $(Y_{j11},...,Y_{jKS}) \in H_j$, $k \in K^o$; (ii) consumers value these commodities: $K^o \subseteq K^c$; (iii) every consumer *i* holds a positive quantities of at least one commodity in K^o; and (iv) consumer $i=1$ holds positive quantities of all commodities in K^o.◊

Finally, we need to ensure the boundedness of the aggregate production possibility set.

Assumption P3 (production set): Every firm *j* chooses techniques within the production set H_j that is (i) convex; (ii) nonempty with possibility of inaction: $0 \in H_j$; and (iii) such that $Q = \{(y_{11},...,y_{KS}) \mid y_{ks} = \sum_j Y_{jks} + \kappa_{ks} \sum_i g_{ik}\omega_{ik}; (Y_{1j},...,Y_{Sj}) \in H_j, \text{all } j; 0 \leq g_{ik} \leq 1, all\ ik\}$, the aggregate production set, is bounded from above.◊

Proposition 4 (existence and efficiency of general equilibrium): *Consider the general equilibrium model (6.2,6.11-6.14),and let utility functions and production sets satisfy assumptions U4 and P3, respectively, while endowments meet assumption E1. Then,*

1. *an equilibrium exists,*
2. *every such equilibrium is Pareto-efficient,*
3. *consumer prices* p_k^f *are nonnegative for all k, and positive for* $k \in K^c$
4. *endowment prices* p_k^o *are positive for all* $k \in K^o$.

Proof.

We specify a convex program, using (a) as utility function:

$$max_{w_{kr} \geq 0, X_{ikr} \geq 0, Y_{jks}\ all\ i,j,k,r,s; g_{ik} \geq 0, all\ i,k} \sum_i \alpha_i \sum_k \sum_r w_{kr} U_{ikr}(X_{ikr} / w_{kr})$$

subject to

(b)

$$\sum_r \hat{P}_{krs} \sum_i X_{ikr} = \sum_j Y_{jks} + \kappa_{ks} \sum_i g_{ik}\omega_{ik} \qquad (p_{ks})$$

$$\sum_r w_{kr} = 1$$

$$g_{ik} \leq 1$$

$$(Y_{j11},...,Y_{jKS}) \in H_j$$

Hence, since the objective is concave (assumption U4(i)-(ii) and Lemma 1(iii), and nonempty and bounded (assumptions P3(ii),(iii)), the program defines a budget surplus:

(c) $$b_i = \sum_j \theta_{ij} \sum_k \sum_s p_{ks} Y_{jks} + \sum_k (\sum_s p_{ks} \kappa_{ks}) g_{ik} \omega_{ik} - \sum_k \sum_r (\sum_s p_{ks} \hat{P}_{krs}) X_{ikr} .$$

Profits are nonnegative because of possibility of inaction (assumption P3(ii)). The value of endowments is nonnegative because of the possibility to set g_{ik} to zero. Hence, following standard arguments (see Ginsburgh and Keyzer, 1997, chapter 3), we can construct a fixed point mapping to adjust welfare weights α on the simplex until no consumer has a surplus. However, the proof that this fixed point is an equilibrium is more intricate than is usually the case because of the possibility of negative prices. We proceed as follows.

First, because welfare weights lie on the simplex, at least one consumer must have positive weight. Since in any optimum of the program $\alpha_i \frac{\partial U_{ikr}}{\partial X_{ikr}} \leq \sum_s p_{ks} \hat{P}_{krs}$, for the commodities k in K_i^c in which this consumer's utility is increasing, his expenditure $\sum_r (\sum_s p_{ks} \hat{P}_{krs}) X_{ikr}$ must be positive (by assumption U4(iii)). Furthermore, balances for characteristics imply that $\sum_r (\sum_s p_{ks} \hat{P}_{krs}) \sum_i X_{ikr} = \sum_j \sum_s p_{ks} Y_{jks} + \sum_s (p_{ks} \kappa_{ks}) \sum_i g_{ik} \omega_{ik}$. Since the commodity is not produced, the first term on the right-hand side is non-positive. Hence, $(\sum_s p_{ks} \kappa_{ks}) g_{ik} \omega_{ik}$ is positive as well. This ensures that consumer 1 who, by assumption E1(iv), owns this good has positive income. He must, therefore, have positive consumption, since otherwise he would have a budget surplus. This in turn means that he must have a positive welfare weight. Hence, all consumer prices p_{ks} for $k \in K^c$ are positive, and therefore all consumers have positive income, and since they have no budget surplus, they have a positive welfare weight. As all balances hold with equality, and no consumer has a surplus, it follows that no one has a deficit, and the welfare program defines weights such that all consumer budgets hold with equality. It is then standard (second welfare theorem) to verify that this welfare optimum in (b) supports a competitive equilibrium with zero transfers.

2. *Pareto efficiency.* Since all welfare weights are positive any Pareto-inefficient allocation would contradict optimality.

3. *Positive consumer prices.* Since U_{ikr} is nondecreasing in X_{ikr}, consumer price p_{kr}^f is non-negative for all commodities: $0 \leq w_{kr} \alpha_i \frac{\partial U_{ikr}}{\partial X_{ikr}} \leq w_{kr} (\sum_s p_{ks} \hat{P}_{krs}) = p_{kr}^f$. Since consumer 1 has positive welfare weight, U_{ikr} is increasing for $k \in K^c$ and hence p_{kr}^f is positive for this k whenever w_{kr} is positive.

4. *Positive endowments prices.* Since $K^o \subseteq K^c$, the price of endowments p_k^o is positive for $k \in K^o$. □

Prices of characteristics are determined by the rates of substitution across outputs, in the absence of free disposal, and not by marginal utilities of consumers. Retailers choose, collectively, the regime they prefer for a particular commodity.

Standard imposed by chain

In the model of proposition 4, standards are only imposed at the moment products reach the consumer. Assertion 6 indicates that firms collectively adhere to the standard but individually, they maximize profits on the basis of the prices of characteristics. As a final extension we differentiate standards by chain. We assume that all products, that are traded externally by the chain are standardized, including intermediate goods not purchased by the consumer, and traded at price p_k^c. Hence, chains trade in commodities with one another, and in characteristics internally. Chain *c* comprises firms J_c whose total net supply q_{ckr} of commodity *k* with label *r* balances with consumer demand:

$$\sum_r \sum_i w_{kr} x_{ikr} = \sum_r \sum_c w_{kr} q_{ckr} , \tag{6.15}$$

while clearing at price p_k^c. Chains buy endowments in quantities m_{ck} that in total balance with availability

$$\sum_c m_{ck} = \sum_i g_{ik} \omega_{ik} \tag{6.16}$$

and clear at prices p_k^o, while within every chain characteristics balance:

$$\sum_r \hat{P}_{krs} w_{kr} q_{ckr} = \sum_{j \in J_c} Y_{jks} + \sum_c \kappa_{ks} m_{ck} . \tag{6.17}$$

The adaptation illustrates that many variants could be considered. On the one hand, we can model external trade in commodities with their certified but inflexible uncertainty profile (or composition). On the other hand, we have the internal trade in characteristics, where the differences in mixes between firms can compensate one another. Yet, we have also seen that it depends on the underlying technology whether this effectively happens, and there will be no scope for substitution if uncertainty itself affects all output in the same way as in (4.3), or if there is contamination among firms, as in (4.9).

Section 7
Conclusion

A label is an effective tool to provide the consumer with information on the physical and moral content of food products. It reduces the asymmetry of information between buyer and seller. It also makes it easier for firms to recover the cost of R&D and realize the returns to scale in design and the network externalities. However, it also plays an important role in circumstances where agents are equally well informed. The main aim of this paper has been to highlight this aspect.

It appears that in a partial equilibrium model with endogenous prevention, a single commodity, a single consumer and without uncertainty in production, the economy can become trapped in an inefficient equilibrium, and illustrated this by means of an example that used second-order Taylor approximation of arbitrary profit and utility functions. Next, we found that this inefficiency cannot arise if the average utility gain from prevention is fixed, or if labeling is effectuated with discrete standards. The first option drops out when we consider heterogeneous consumers. In a partial equilibrium setting with heterogeneous consumers, it reduces to the requirement that the average utility gain should be fixed and common across all consumers, whereas in general equilibrium even this requirement fails to resolve the problem. This option also drops out if utility functions are not additively separable across commodities. Hence, labeling emerges as the only option capable of eliminating inefficiency under all circumstances considered. By creating the possibility of co-existence of several varieties, it enables the consumer to reach the convex hull of points in the choice set that are not attainable under perfect information without labeling.

Endogenous prevention was also seen to generate a non-rivalry across states that could be avoided through labeling, but re-emerged when we considered heterogeneous consumers. We also found that in a general equilibrium model it could eliminate the possibility of negative income, that was due to the inherent lack of free disposal of undesirable characteristics.

We also studied a deterministic model of optimal product design. In this case, a possible (Pareto)-inefficiency arises only when we have to deal with multiple consumers facing the same bundle of characteristics. Then, we have to invoke utility functions that are additively separable over commodities, and a common valuation of characteristics across consumers as a factor on the consumer specific-utility, jointly with a scale independent cost of delivering the bundle of characteristics, or discrete designs. Clearly, the first requirement is highly restrictive. Therefore, also with respect to the optimal selection of product characteristics, and in the same way as for unsafety, discrete standards broaden the range of varieties and come out as the only way to avoid inefficiency.

Appendix
Coefficients of Example 2.

$S = 4$, $p^c = 20.0430$,

b = [.100000 .235714 .300000 .364286],

$$B = \begin{bmatrix} .0144 & .0048 & -.0048 & -.0144 \\ .0048 & .0500 & -.0016 & -.0532 \\ -.0048 & -.0016 & .0340 & -.0276 \\ -.0144 & -.0532 & -.0276 & .0952 \end{bmatrix},$$

$\alpha = [3.0473 \quad 3.0232 \quad 2.9991 \quad 2.9750]$,

$a = [20.066 \quad 20.061 \quad 20.055 \quad 20.050]$,

$A = [.0125 \quad .0250 \quad .0375 \quad .0500]$.

Figure 1 shows the value of welfare $W(x) - 3000.$ on the y-axis, against consumption x.

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The Centre for World Food Studies (Dutch acronym SOW-VU) is a research institute related to the Department of Economics and Econometrics of the Vrije Universiteit Amsterdam. It was established in 1977 and engages in quantitative analyses to support national and international policy formulation in the areas of food, agriculture and development cooperation.

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