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Introducing aggregate risk in an orthodox intertemporal Arrow-Debreu model:

what happens to saving and investment?

by

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Abstract

We study the effect of aggregate risk on saving and investment in an illustrative two-period general equilibrium model with contingent contracts (possibly constrained) in the Arrow-Debreu tradition and rather standard functional forms. Risk is modelled in the form of independent sector-specific endowment shocks. Household classes maximize expected utility. In a partial setting this specification of preferences and behaviour would imply that risk leads to precautionary saving, with the largest tendency for the poorest classes. Simulations with the two-period model show that precautionary saving persists also in the general equilibrium context, in spite of the equilibrium price variability and the smoothing provided by contingent contracts. However, the precautionary saving effect is drastically reduced if investment itself is stochastic, more precisely if its impact is positively correlated to the aggregate risk in the economy. Surprisingly, savings hardly react to constraints on contingent transactions. Only when the assumption of rational expectations is dropped and household classes are assumed to be optimistic or pessimistic in the perception of their expected utility, bounds on contingent transactions have a significant impact on saving rates. Furthermore, the simulations show that in an orthodox Arrow-Debreu context with aggregate risk, liquidity constraints are not necessarily detrimental to the poor.

Section 1
Introduction[1]

One may distinguish three different approaches to incorporate uncertainty in a general equilibrium model. The traditional approach of Arrow (1953) and Debreu (1959) assumes the existence of a complete set of contingent markets which allows the actors to formulate mutually consistent Pareto-optimal intertemporal plans of consumption and production. The GEI-approach (General Equilibrium with Incomplete asset markets) maintains the assumption of mutually consistent intertemporal plans but drops the assumption of complete availability of contingent markets, leading to loss of Pareto-optimality, as explained carefully in Magill and Quinzii (1996). The temporary equilibrium approach, analyzed theoretically in Grandmont (1977), takes a further step away from Arrow and Debreu by abandoning the assumption of mutual consistency of the future part of intertemporal plans and, therefore, requiring annual revisions of the plans.

The literature on the three approaches is largely theoretical, at least for models with more than one commodity and more than one actor. Applied general equilibrium models with several actors and commodities usually do not consider uncertainty explicitly, not even when they emphasize the role of the financial sector, as Rosensweig and Taylor (1990) and Bourguignon, Branson and De Melo (1992) do. These applied models consist of a sequence of annual static models, linked by resource updates, in which actors maximize utility under perfect foresight just for the current period.

This paper is meant as a step in bridging the gap between theoretical and applied work, by showing quantitatively how aggregate risk influences the outcomes of an orthodox intertemporal Arrow-Debreu model with several actors and commodities. The Arrow-Debreu approach seems a better point of departure for such an analysis than either the GEI approach or the temporary equilibrium approach since it avoids the necessity to specify the structure of the incompleteness of existing contingent markets, a task which would be rather demanding on its own due to the large variety of formal and informal financial contracts prevailing in each society. Instead of restricting the available set of contingent markets, we will limit the ‘ideal’ financing possibilities of the Arrow-Debreu approach by imposing liquidity constraints on the actors. The liquidity constraints are specified separately for transactions across periods (borrowing constraints) and across events (insurance constraints).

[1] The author thanks Chris Elbers and Jan-Willem Gunning for their comments on earlier versions, and Michiel Keyzer for his advice in the implementation of the algorithm.

We study an illustrative two-period closed general equilibrium model with intertemporal saving and investment decisions. Actors are uncertain about their period-2 endowments, although aware of the probability distributions of random events. The probability distributions are assumed to be continuous, implying that the number of possible future states is infinite. The uncertainty is aggregate in the sense that the random events do not only affect the endowments of individual actors but also the endowments of the economy as a whole. Hence, period-2 prices are stochastic. Behaviour follows from maximization of expected utility. The latter may be cast in terms of subjective probability estimates, different from the actual common assessment of probabilities. This option implies that the expectations are 'correct' in the sense of Magill and Quinzii (1996) but need not be perfectly rational: actors can be optimistic or pessimistic in the assessment of their expected utility. Deliberately, the model has rather standard functional forms. The data used are typical for a country in which income differences are pronounced and revenues of the household classes and government depend disproportionally on specific production sectors. The model is solved by applying a stochastic optimization method proposed by Ermoliev and Keyzer (1998).

In a partial context, under rather standard assumptions about utility maximization and functional forms, risk leads to additional savings, and this precautionary motive is stronger for poor than for rich actors. At the same time, the concern for the future raises expected future prices which leads to higher investment, although possibly partly offset by uncertainty of the investment impact itself. The central question of the paper is to which extent mean-preserving risk indeed leads to higher saving and investment in a general equilibrium context in which also prices and contingent contracts act as smoothing mechanism. Possibly, the outcomes differ between poor and rich classes, especially when liquidity constraints are tight. In this respect, we will consider not only average outcomes but also the spread in outcomes.

Section 2 describes the model, assuming rational expectations of the actors. Section 3 analyzes what can be said theoretically about the behaviour of the actors under uncertainty. In section 4 the model is reformulated in the Negishi-format to which the solution algorithm actually applies. Sections 5 and 6 describe simulation experiments under perfect foresight and widespread risk, respectively. Section 7 discusses the impact of alternative risk aversion parameters. Sections 8 and 9 extend the scope of the simulations by abandoning the rationality of the actors and tightening the constraints on borrowing and insurance. Finally, section 10 summarizes the findings. Annex A contains the list of symbols used, annex B discusses the relation between precautionary savings and income level, and annex C gives the full list of outcomes of the base simulation under widespread risk.

Section 2
Description of the economy

We consider a closed economy with four agents, i.e. three household classes and government, and distinguish two periods. Eight commodities are supplied and traded on competitive markets. In each period prices adjust to clear the markets. Each household class is simultaneously producer and consumer, and so is government. In period 1 commodity supply consists merely of exogenous endowments. Also in period 2 the agents dispose of exogenous endowments but this time the endowments are stochastic implying that their realization is not known before period 2. Furthermore, supply in period 2 can be increased by investing in period 1. Transfers are exogenous and expressed in terms of a commodity basket. Households derive utility from commodity consumption. At the beginning of period 1 they decide about consumption and investment levels such that expected intertemporal utility is maximized, while satisfying the intertemporal budget constraint. Hence, a budget deficit in one of the periods must be compensated for in the other period. Utility is assumed to be intertemporally additive. Government behaves in the same way as a household.[2]

The following notation is used.[3] There are m actors, indexed i, and r commodities, indexed k. Hence, m = 4 and r = 8. The time subscript is t. In period t actor i's consumption is represented by r-vector x_{it}, the endowments by r-vector ω_{it}, the investment level by scalar d_{it}, net transfer receipts by scalar $\bar{t}_{it}$, income by scalar h_{it} and the budget deficit by scalar V_{it}. The r-vector p_t stands for the prices in period t. The symbols u_{it} denote the instantaneous utility functions. Uncertainty in period 2 is reflected by the random event ε_2 which is an N-vector of random sources, indexed n, with joint density function f. The uncertainty comes from sources beyond direct control of the actors and may be attributed e.g. to weather fluctuation or political instability. Assuming rational expectations (to be relaxed afterwards), household behaviour is then represented as maximization of expected utility subject to the budget constraint and liquidity constraints. In equations, with shadow prices of the constraints between brackets:

$$\max_{x_{i1}, x_{i2}(\varepsilon_2), d_{i1} \geq 0} \quad u_{i1}(x_{i1}) + \int u_{i2}(x_{i2}(\varepsilon_2)) f(\varepsilon_2) d\varepsilon_2 \qquad (2.1)$$

$$\text{s.t.} \quad V_{i1} + \int V_{i2}(\varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \leq 0 \qquad (\lambda_i)$$

$$V_{i1} \leq Z_{i1} \qquad (\mu_{i1})$$

[2] At least, in the current version. Alternatively, government consumption and investment levels may be set exogenously, with total household taxes as balancing budget item.
[3] Annex A contains the full list of symbols.

$$V_{i2}(\varepsilon_2) \le Z_{i2} - V_{i1} \qquad (\mu_{i2}(\varepsilon_2))$$

in which

$$h_{i1} = p_1(\overline{\omega}_{i1} + \bar{t}_{i1}\vartheta)$$

$$V_{i1} = p_1(x_{i1} + \eta_i d_{i1}) - h_{i1}$$

$$Z_{i1} = \ell_{i1} h_{i1}$$

$$Z_{i2} = \ell_{i2} h_{i1}$$

$$\omega_{i2} = \overline{\omega}_{i2}(\varepsilon_2) + g_i(d_{i1}, \varepsilon_2)$$

$$h_{i2}(\varepsilon_2) = p_2(\varepsilon_2)(\omega_{i2} + \bar{t}_{i2}\vartheta)$$

$$V_{i2}(\varepsilon_2) = p_2(\varepsilon_2)(x_{i2}(\varepsilon_2) + \eta_i \overline{d}_{i2}) - h_{i2}(\varepsilon_2)$$

Household decisions and prices in period 2 are not deterministic but depend on the random event, since its outcomes are known in period 2. This dependence is made explicit by writing the variables as functions of the random event. The coefficient-vector η_i specifies the commodity requirements per unit of investment, and the vector-valued function g_i measures the impact of investment on next year's endowments. This impact may depend on the random event. The symbol ϑ (r-vector) is the commodity basket in which transfers are expressed, and the scalars ℓ_{it} determine the exogenously imposed liquidity constraints faced by actor i. These constraints may reflect e.g. deficiencies in the country's banking or insurance system leading to levels of financial transactions that are sub-optimal from the welfare-economic point of view. Period-2 investment levels are set exogenously since, otherwise, they would become zero in this two-period model.

Households can increase their expected utility by (1) allocating consumption within periods, (2) allocating consumption across periods, (3) allocating consumption across possible events, and (4) engaging in investment activities. The budget constraint must merely hold in terms of expectations, and not for each random event. This formulation specifies the possibility of insurance (exchange of risk) across actors. At the beginning of period 1 the actor does not know the realization in period 2 but he commits himself to a (positive or negative) resource transfer of $V_{i2}(\varepsilon_2)$, if event ε_2 occurs. The net insurance component of this transfer can be calculated as the difference with the expected transfer, i.e.

$$V_{i2}(\varepsilon_2) - \int V_{i2}(\varepsilon_2{}')\, f(\varepsilon_2{}')\, d\varepsilon_2{}' \ .$$

When the budget constraint is satisfied, this amount is equal to $V_{i2}(\varepsilon_2) + V_{i1}$. Therefore, bound Z_{i2} in the period-2 liquidity constraint is a bound on insurance whereas the period-1 bound Z_{i1} is a bound on borrowing. Both bounds are expressed as share of income in period 1.

The term ‘insurance’ is used here in the general sense of shifting risk across actors. Although this description, in the words of Arrow,[4] indicates the ‘very essence of insurance’, it is broader than the use of the word in everyday language, in which insurance is reserved for compensation payments in case of adverse events. Here, it covers not only the side of the insured but also the opposite side, i.e. the side of the insurer who gets part of the gains if events are favorable. Furthermore, since we consider aggregate risk, both sides may be hit by the same event and still agree upon a contingent payment, depending on who is most affected in terms of utility. The ‘insurance premium’ is not modelled explicitly but subsumed in the set of mutual contingent obligations agreed upon in advance.

Prices are taken as given, and follow from market clearing which means that, in spite of the uncertainty, the actors agree on the existence of clearing prices, also for period 2:

$$\begin{array}{lll} p_1 \geq 0 & \perp & \sum_i x_{i1} + \sum_i \eta_i d_{i1} \leq \sum_i \overline{\omega}_{i1} \\ p_2(\varepsilon_2) \geq 0 & \perp & \sum_i x_{i2}(\varepsilon_2) + \sum_i \eta_i \overline{d}_{i2} \leq \sum_i \omega_{i2} \end{array} \qquad (2.2)^5$$

As opposed to the individual budget constraints, market clearing is required for each random event in period 2. All contingent markets exist, in the tradition of Arrow (1953) and Debreu (1959). Transaction costs are neglected. The existence of the contingent markets allows determination of the endogenous insurance amounts in each event. Market clearing implies also that the total budget constraint of the country is satisfied at these prices. Thus, in each event the actors can indeed find counterparts for their resource transfers.

In addition to program (2.1) and conditions (2.2), the model is completed with a price normalization rule (affecting prices in both periods simultaneously). Such a rule does not influence the real side due to the absence of non-homogeneities. The rule adopted here sets the price of the commodity basket in the first period 1 at one:

$$p_1 \vartheta = 1 \qquad (2.3)$$

Functional forms are assumed to be rather standard. The instantaneous utility function is an iso-elastic transformation of a Stone-Geary function, preceded by an exogenous time preference parameter. The random sources are independently and uniformly distributed, with

[4] See Arrow (1965), page 45.

[5] The symbol $\perp$ indicates complementarity: $x \geq 0 \perp y \geq 0$ for the vectors x and y means that $x \geq 0, y \geq 0$ and $xy = 0$.

marginal density functions f^n. Endowment risk is sectoral. For each sector the uncertainty is linked by a pointer $\iota(k)$ to, at most, one of these sources, with the option of positive correlations across sectors. Investment is specified as a production function with decreasing returns to scale that needs inputs in period 1 and yields output in period 2. Two versions are distinguished. In one version the investment impact is subject to the same random events as the exogenous endowments, in the other version the investment impact is non-stochastic. More specifically, the following functional forms are used:

$$\overline{\omega}_{ik2}(\varepsilon_2) = \overline{\omega}^{\circ}_{ik2}(1+\varepsilon_{n2}) \quad \text{with } n = \iota(k) \text{ and } \overline{\omega}^{\circ}_{ik2} \in R \tag{2.4}$$

$$f^n(\varepsilon_{n2}) = 1/(2\tau_n) \quad \text{for } \varepsilon_{n2} \in [-\tau_n, \tau_n] \text{ and } 0 \text{ elsewhere} \tag{2.5}$$

with $0 < \tau_n < 1$ and small enough to ensure that total endowments of each commodity are always sufficient to satisfy committed demand whereas $f(\varepsilon_2) = \prod_n f^n(\varepsilon_{n2})$

$$u_{it}(x_{it}) = e^{-\delta(t-1)} \; v(w_i(x_{it}); \sigma) \tag{2.6}$$

$$\text{with} \quad v(w;\sigma) = \begin{cases} (w^{1-1/\sigma} - 1)/(1-1/\sigma) & \text{for } \sigma > 0, \sigma \neq 1 \\ \log(w) & \text{for } \sigma = 1^{6} \end{cases}$$

$$w_i(x_{it}) = \prod_k (x_{ikt} - \gamma_{ik})^{\beta_{ik}} \qquad x_{ikt} \geq \gamma_{ik} + s \quad \text{(all i,k,t)}$$

in which $\delta, \beta_{ik}, \gamma_{ik} \geq 0$, $\sum_k \beta_{ik} = 1$ and s a very small, positive value

$$g_i(d_{i1}, \varepsilon_2) = \zeta_i d_{i1}{}^{\nu} \, \overline{\omega}_{i2}(\varepsilon_2) \tag{2.7a}$$

$$\text{or} \qquad \zeta_i d_{i1}{}^{\nu} \, \overline{\omega}^{\circ}_{i2} \tag{2.7b}$$

with $\overline{\omega}_{i2}(\varepsilon_2) \in R^r$, $\overline{\omega}^{\circ}_{i2} \in R^r$, $\zeta_i \in$ R, $\nu \in$ R , $\zeta_i > 0$ and $0 < \nu < 1$.

With respect to terminology, we did already introduce the term *aggregate risk* for risk that affects the total endowments of a sector, and not just the allocation of endowments over the actors. In addition, we will use the term *widespread risk* to indicate random events in which all sectors of the economy are affected, albeit possibly to a different degree. Furthermore, investment equation (2.7a) will be called the specification with *stochastic investment impact*, and equation (2.7b) the specification with *deterministic investment impact*.

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[6] Then $v(w;1) = \lim_{\sigma \to 1} v(w;\sigma)$.

With the model defined here we will study how risk influences the saving and investment decisions of the actors. First, we will analyze what can be said theoretically, i.e. purely based on the structure of the model and without quantitative application.

Section 3
Effects of risk on intertemporal decisions: theoretically

This section starts with a discussion of the first-order conditions that characterize the decisions of the actors. Then, we study what can be derived from these conditions about the impact of uncertainty on period-1 saving and investment at given prices, hence in a partial context. Finally, we turn to the full general equilibrium model.

Actor behaviour follows from the first-order conditions of utility maximization program (2.1).[7] In each period and for each event actor i chooses his consumption package such that the marginal utility of consumption per unit of expenditures is equal across commodities:

$$\frac{\partial u_{i1}/\partial x_{ik1}}{p_{k1}} = \lambda_i + \mu_{i1} + \int \mu_{i2}(\varepsilon_2) f(\varepsilon_2)\, d\varepsilon_2 \qquad \text{for } x_{ik1} > \gamma_{ik} + s \qquad (3.1)$$

$$\frac{\partial u_{i2}/\partial x_{ik2}(\varepsilon_2)}{p_{k2}(\varepsilon_2)} = \lambda_i + \mu_{i2}(\varepsilon_2) \qquad \text{for } x_{ik2}(\varepsilon_2) > \gamma_{ik} + s \qquad (3.2)$$

Equation (3.2) clearly reveals the importance of insurance: the marginal utility per unit of expenditures is equal across all events in period 2, viz. equal to λ_i, if none of the insurance constraints is binding. If neither the borrowing constraint is binding, the equality of the marginal utility per unit of expenditures even extends to period 1. In this case, the intertemporal marginal rates of substitution of actor i are equal to the market price ratios which implies that changing the consumption allocation cannot lead to a utility gain of one actor without reducing the utility of an other actor. In other words, with non-binding liquidity constraints the equilibrium solution is Pareto-optimal.

For the functional form of utility specified in (2.6), i.e. the iso-elastic transformation of a Stone-Geary function, the first-order conditions imply (i) that within each period and at each event, consumption follows a linear expenditure system with uncommitted budget shares β_{ik} and commitments γ_{ik}, and (ii) that, if liquidity constraints are not binding, the elasticity of substitution of consumption[8] across periods is constant and equal to $-\sigma$.

The first-order conditions with respect to investments show that the actors set their investment level such that expected marginal revenues equal marginal costs. In this two-period model all benefits of investments must be reaped in period 2. The costs are incurred in

[7] The derivation of the first-order conditions is specified in Van Veen (forthcoming), based on a theorem of Hestenes as formulated in chapter 8 of Takayama (1974).

[8] Defined in terms of the Stone-Geary aggregate $w_i(x_{it})$.

period 1 and consist of market costs and, possibly, actor-specific costs in case of binding liquidity constraints:

$$\int (1+\mu_{i2}(\varepsilon_2)/\lambda_i)\, p_2(\varepsilon_2) \frac{\partial g_i(d_{i1},\varepsilon_2)}{\partial d_{i1}} f(\varepsilon_2)\, d\varepsilon_2 = \qquad (3.3)$$
$$p_1\eta_i\ [1+(\mu_{i1}/\lambda_i)+\int(\mu_{i2}(\varepsilon_2)/\lambda_i) f(\varepsilon_2) d\varepsilon_2] \qquad \text{for } d_{i1}>0$$

The concavity of the function g_i in d_{i1} implies that the investment level is positively related to period-2 prices, and negatively to period-1 prices and the borrowing constraint. The effects of tighter insurance constraints are not clear, since they influence both sides of (3.3). The concavity of g_i also implies that the expected average net return must be larger than the expected marginal net return on investments, hence larger than zero. For functional form (2.7) the expected average net return can be derived as $(1/\nu)-1$ times the expected costs, at least when the shadow prices of the liquidity constraints are zero.

Maximization of expected utility and concavity of the utility function characterize a risk-averse attitude of the actor. In other words, the expected utility of consumption is valued at most equal to the utility of expected consumption, as shown by Jensen's inequality:[9]

$$\int u_{i2}(x_{i2}(\varepsilon_2))\, f(\varepsilon_2)\, d\varepsilon_2 \le u_{i2}(\int x_{i2}(\varepsilon_2)\, f(\varepsilon_2)\, d\varepsilon_2) \qquad (3.4)$$

The degree of risk aversion depends on the curvature of the instantaneous utility function. Defining the curvature as the absolute value of the elasticity of marginal utility with respect to consumption leads to the Arrow-Pratt measure of relative risk aversion, written here in terms of a consumption aggregate $X_{it}(x_{it})$:

$$-\frac{X_{it} u''_{it}(X_{it})}{u'_{it}(X_{it})} \qquad (3.5)$$

For the functional form specified in (2.6) and using the Stone-Geary aggregate $w_i(x_{it})$, the Arrow-Pratt measure of relative risk aversion is constant and equals $1/\sigma$. Hence, a higher value of σ means a reduced curvature and, therefore, lower relative risk aversion. It is a well-known property (or problem) of intertemporal additivity in an expected utility setting that the degree of intertemporal substitution is inversely related to the degree of relative risk aversion,[10] as reflected here in the double role of the parameter σ. Constant relative risk

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[9] See e.g. Stirzaker (1994), page 99. Strictly speaking, the formulation in (3.4) requires the function $x_{i2}(\varepsilon_2)$ to be one-to-one, allowing transformation of the density function.

[10] See e.g. Deaton (1992), page 20.

aversion is consistent with (although not necessarily following from) the generally accepted assumption of declining absolute risk aversion. [11]

Now we address the impact of risk on saving and investment. Leland (1968) shows that the assumption of risk aversion alone is not sufficient to draw conclusions about the reactions of actors to increasing uncertainty and, in particular, to guarantee a positive precautionary demand for saving. The latter should be ensured by additional assumptions, notably intertemporal additive utility and $u_{it}'''(X_{it}) > 0$. Positivity of this third-order derivative is an acceptable assumption since it follows immediately from declining absolute risk aversion. Indeed, functional form (2.6) satisfies this condition. Leland's derivation uses a separate budget constraint for each uncertain event, thus seemingly excluding the possibility of insurance. However, the same argument holds under incomplete insurance. Therefore, it is also valid for program (2.1), in which the economy as a whole cannot insure itself.

Without taking into account the concrete functional form of the utility function, it does not seem possible to relate the precautionary saving motive to the income of the actors. But for equation (2.6), as mentioned earlier a rather standard functional form, a negative impact of income on precautionary savings can be derived. Annex B gives the details, in a context similar to Leland's. Hence, for poor actors one may expect a stronger saving reaction to risk than for rich actors.

With respect to the investment reaction to risk, such a direct distinction between poor and rich cannot be made. Differences in reactions across actors are not related to differences in income levels or utility preferences, but to differences in investment technology and shadow prices of liquidity constraints. If we neglect the latter, condition (3.3) simplifies into

$$\int p_2(\varepsilon_2)\frac{\partial g_i(d_{i1},\varepsilon_2)}{\partial d_{i1}} f(\varepsilon_2)\, d\varepsilon_2 = p_1\eta_i \qquad (3.6)$$

For functional form (2.7a) :

$$\frac{\partial g_i(d_{i1},\varepsilon_2)}{\partial d_{i1}} = \zeta_i \nu\, d_{i1}{}^{\nu-1}\, \overline{\omega}_{i2}(\varepsilon_2)$$

Substitution in (3.6) gives:

$$d_{i1}{}^{1-\nu} = \frac{\zeta_i \nu}{p_1\eta_i}\int p_2(\varepsilon_2)\,\overline{\omega}_{i2}(\varepsilon_2)\, f(\varepsilon_2)\, d\varepsilon_2 \qquad (3.7)$$

[11] See e.g. Arrow (1965), page 35, and Deaton (1990), page 64. The degree of absolute risk aversion is defined as $-u_{it}''(X_{it})/u_{it}'(X_{it})$.

Since $0 < \nu < 1$, the level of investment is positively related to the value of the integral on the righthandside, i.e. to the expected value of endowment vector $\overline{\omega}_{i2}(\varepsilon_2)$. This relation leads to three conclusions, holding also at given non-zero shadow prices of the liquidity constraints. First, a mean-preserving increase in endowment uncertainty does not change the investment level, if prices are considered as given. Secondly, a mean-preserving change in period-2 price uncertainty does not change the investment level if the endowment vector in the integral is non-stochastic, hence if the investment impact is non-stochastic. Thirdly, if the investment impact is stochastic according to (2.7a), the effect of a simultaneous mean-preserving introduction of endowment ánd price uncertainty on investment depends on the covariance between endowment levels and prices. In the general equilibrium context the covariance is negative and, therefore, the expected value of the product of $p_2(\varepsilon_2)$ and $\overline{\omega}_{i2}(\varepsilon_2)$ is less than the product of the expected values of $p_2(\varepsilon_2)$ and $\overline{\omega}_{i2}(\varepsilon_2)$,[12] which means that the investment level is reduced.

This brings us to the full general equilibrium model. The partial analyses above show that, at given prices, mean-preserving risk affects only the consumer allocations via precautionary savings. In the full model they subsequently influence also prices and investment decisions until a new equilibrium has been found, possibly with changes in the severity of the liquidity constraints. In response to the precautionary savings, prices in period 2 will go up compared to period 1, leading to positive investment reactions. When the function g_i itself is indeed stochastic, there is also an inverse investment reaction, due to the negative correlation between price and endowment uncertainty. In the end, since the model is closed, the saving-investment balance must be restored in each period.

However, it is not a priori clear to which extent the period-1 totals go up and how the saving deficits of the actors will adjust. The theory is not conclusive in this respect, and quantitative simulations are required to explore possible consequences. The simulations will be performed from section 5 onwards. First, the model will be rewritten in a different format which makes it possible to actually compute solutions under uncertainty.

[12] The covariance formula specifies that $E(p_2\overline{\omega}_{i2}) = (E\,p_2)(E\,\overline{\omega}_{i2}) + \mathrm{Cov}(p_2, \overline{\omega}_{i2})$, in which E stands for expectation and Cov for covariance.

Section 4
The model in Negishi-format

As long as uncertainty is described via a limited set of discrete states, the model of section 2, i.e. program (2.1) with complementarity conditions (2.2) and price normalization rule (2.3), can be solved in a rather standard way. But when a continuous density function is applied or a large number of discrete states is distinguished, the solution process becomes cumbersome due to the need to optimize over the functions $x_{i2}(\varepsilon_2)$ simultaneously with the iteration over the stochastic price variables $p_2(\varepsilon_2)$ for market clearing. To circumvent these problems Ermoliev and Keyzer (1998) propose to write this kind of models in Negishi-format and to decompose the intertemporal problem. Then the solution can be obtained by computing stochastic quasi-gradients, a technique from the field of stochastic optimization.

The model in section 2 is specified in terms of decisions of individual actors whose behaviour is coordinated by market prices. This formulation is often referred to as the excess-demand format. Negishi (1960) shows that a general equilibrium model can equivalently be expressed in a centralized format, i.e. by a central agency that maximizes a social welfare function, provided that it adjusts the welfare weights such that all individual budget constraints are satisfied. This central agency is not an agent itself but merely a 'processor of information': it does not impose its own views and respects all individual preferences and property rights. The Negishi format is a welfare program with feedback.

The welfare program maximizes the social welfare function, at given positive welfare weights α_i, subject to the commodity balances of each period and event and subject to the liquidity constraints. In equations, with shadow prices of the constraints between brackets:

$$\max_{x_{i1}, x_{i2}(\varepsilon_2), d_{i1} \geq 0 \text{ for all } i} \quad \sum_i \alpha_i \left[u_{i1}(x_{i1}) + \int u_{i2}(x_{i2}(\varepsilon_2)) f(\varepsilon_2) d\varepsilon_2 \right] \qquad (4.1)$$

s.t. $\sum_i x_{i1} + \sum_i \eta_i d_{i1} \leq \sum_i \overline{\omega}_{i1}$ $\quad (p_1)$

$\sum_i x_{i2}(\varepsilon_2) + \sum_i \eta_i \overline{d}_{i2} \leq \sum_i \omega_{i2}$ $\quad (p_2(\varepsilon_2))$

$V_{i1} \leq Z_{i1}$ $\quad (\breve{\mu}_{i1})$ $\quad$ for all i

$V_{i2}(\varepsilon_2) \leq Z_{i2} - V_{i1}$ $\quad (\breve{\mu}_{i2}(\varepsilon_2))$ $\quad$ for all i

in which $h_{i1} = \tilde{p}_1(\overline{\omega}_{i1} + \bar{t}_{i1}\vartheta)$

$V_{i1} = \tilde{p}_1(x_{i1} + \eta_i d_{i1}) - h_{i1}$

$$Z_{i1} = \ell_{i1} h_{i1}$$

$$Z_{i2} = \ell_{i2} h_{i1}$$

$$\omega_{i2} = \overline{\omega}_{i2}(\varepsilon_2) + g_i(d_{i1}, \varepsilon_2)$$

$$h_{i2}(\varepsilon_2) = \tilde{p}_2(\varepsilon_2)(\omega_{i2} + \bar{t}_{i2}\vartheta)$$

$$V_{i2}(\varepsilon_2) = \tilde{p}_2(\varepsilon_2)(x_{i2}(\varepsilon_2) + \eta_i \bar{d}_{i2}) - h_{i2}(\varepsilon_2)$$

and with extra iterations such that $\tilde{p}_2(\varepsilon_2) = p_2(\varepsilon_2)$

In the feedback component welfare weights α_i are adjusted such that

(i) budgets are satisfied (relative adjustment):

$$V_{i1} + \int V_{i2}(\varepsilon_2) f(\varepsilon_2) d\varepsilon_2 = 0 \qquad \text{for all } i \tag{4.2}$$

(ii) the price of the commodity basket in period 1 equals one (normalization):

$$p_1 \vartheta = 1 \tag{4.3}$$

whereas, furthermore, the period-1 price parameters are kept equal to the shadow prices:

$$\tilde{p}_1 = p_1 \tag{4.4}$$

The shadow prices of the commodity balances in the welfare program are the market prices. These prices should coincide with the price parameters in the liquidity constraints. Therefore, they must be equalized in additional loops. The computational advantage of the Negishi-format compared to the excess-demand format is that it requires iteration over the deterministic welfare weights instead of iteration over the stochastic period-2 prices. The latter follow as shadow prices from the welfare program. However, optimization still requires simultaneous optimization over period-1 variables and period-2 functions. Decomposition of the welfare program provides a further computational advantage. Therefore, (4.1) is written as follows:

$$\max_{x_{i1}, d_{i1} \geq 0 \text{ for all } i} \quad \sum_i \alpha_i u_{i1}(x_{i1}) + \int W(x_1, d_1; Z_2, \alpha, \tilde{p}_1, \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \tag{4.1a}$$

s.t. $\sum_i x_{i1} + \sum_i \eta_i d_{i1} \leq \sum_i \overline{\omega}_{i1}$ $\qquad (p_1)$

$V_{i1} \leq Z_{i1}$ for all i $\qquad (\breve{\mu}_{i1})$

with $h_{i1} = \tilde{p}_1(\bar{\omega}_{i1} + \bar{t}_{i1}\vartheta)$

$Z_{i1} = \ell_{i1} h_{i1}$

$Z_{i2} = \ell_{i2} h_{i1}$

$V_{i1} = \tilde{p}_1(x_{i1} + \eta_i d_{i1}) - h_{i1}$

and in which $W(x_1, d_1; Z_2, \alpha, \tilde{p}_1, \varepsilon_2) =$

$$\max_{x_{i2}(\varepsilon_2) \geq 0 \text{ for all } i} \sum_i \alpha_i u_{i2}(x_{i2}(\varepsilon_2)) \tag{4.1b}$$

s.t. $\sum_i x_{i2}(\varepsilon_2) + \sum_i \eta_i \bar{d}_{i2} \leq \sum_i \omega_{i2}$ $\quad (p_2(\varepsilon_2))$

$V_{i2}(\varepsilon_2) \leq Z_{i2} - V_{i1}$ $\quad (\breve{\mu}_{i2}(\varepsilon_2))$ for all i

with $V_{i1} = \tilde{p}_1(x_{i1} + \eta_i d_{i1}) - h_{i1}$

$\omega_{i2} = \bar{\omega}_{i2}(\varepsilon_2) + g_i(d_{i1}, \varepsilon_2)$

$h_{i2}(\varepsilon_2) = \tilde{p}_2(\varepsilon_2)(\omega_{i2} + \bar{t}_{i2}\vartheta)$

$V_{i2}(\varepsilon_2) = \tilde{p}_2(\varepsilon_2)(x_{i2}(\varepsilon_2) + \eta_i \bar{d}_{i2}) - h_{i2}(\varepsilon_2)$

and with extra iterations such that $\tilde{p}_2(\varepsilon_2) = p_2(\varepsilon_2)$

The period-2 program is solved at given period-1 consumption and investment levels, at given insurance bounds, at given welfare weights, at given period-1 price parameters and at given random events. The full model is given by (4.1a), (4.1b), (4.2), (4.3) and (4.4). Neither program (4.1a) nor program (4.1b) has functions as argument. Therefore, first-order conditions (equivalent to those in section 3) can be derived with the standard Kuhn-Tucker theorem.

The decomposition suggests a solution method with three loops: an outer loop over α and $\tilde{p}_1$, a middle loop over x_1 and d_1, and an inner loop over $x_{i2}(\varepsilon_2)$ and $\tilde{p}_2(\varepsilon_2)$. Ermoliev and Keyzer argue that the inner loop does not require convergence of the sequence of Monte-Carlo drawings at each value of (x_1, d_1). Instead, they propose to use a Stochastic Quasi-Gradient method (SQG) which allows for simultaneous updates of (x_1, d_1) and random drawings in the inner loop. The actual application of this approach to the current problem is described in Van Veen (forthcoming). Here, we concentrate on economic aspects of the model.

Section 5
Simulation outcomes under perfect foresight

In this section and the following ones we present quantitative simulations to study the effects of mean-preserving random endowment shocks on saving and investment decisions of the different actors. In the simulations the behavioural parameters of the actors are largely the same, notably time preference δ, parameter of intertemporal substitution σ (at the same time measuring risk aversion) and the marginal rate of return on investments ν. Also for the uncommitted budget shares β_{ik} uniform values are used, at least for the household classes. These similarities make it possible to attribute differences in outcomes to differences in levels of endowments and, hence, to differences in wealth.

In this respect, it is important to note that functional form (2.6) allows for an equivalent interpretation of utility program (2.1) in per capita terms, provided that one assumes that population sizes $\overline{n}_i$ are equal in both periods. Denoting the per capita utility function by $\ddot{u}_{it}$ and per capita consumption by $\ddot{x}_{it}$, the following relation holds between per capita utility and absolute utility:

$$\ddot{u}_{it}(\ddot{x}_{it};\sigma) = \qquad (5.1)$$

$$\begin{cases} (1/\overline{n}_i)^{1-1/\sigma} u_{it}(x_{it}) + e^{-\delta(t-1)} \dfrac{(1/\overline{n}_i)^{1-1/\sigma} - 1}{1-1/\sigma} & \text{for } \sigma > 0, \sigma \neq 1 \\ u_{it}(x_{it}) - e^{-\delta(t-1)} \log(\overline{n}_i) & \text{for } \sigma = 1 \end{cases}$$

This relation presupposes that all parameters of the function $\ddot{u}_{it}$ are the same as the parameters of u_{it}, with the exception of the commitments which are equal to $\gamma_{ik}/\overline{n}_i$ in function $\ddot{u}_{it}$. In the simulations below the per capita commitments are taken to be equal across the household classes. Then, the households have exactly the same preferences and it makes sense to compare the resulting per capita utilities.

The parameters η_i and ζ_i of the investment specification are the only parameters that are not harmonized across households. Together with the period-2 endowment structure they determine the level d_{i1} to which the actors can increase their investments until the expected marginal net return equals zero, or in other words until the expected average net return equals $(1/\nu)-1$ times the expected costs. These parameters are used to bring about differences in investment profitability and, hence, in the period-1 investment rates of the actors in the base run.

The three household classes are rural, urban poor and urban rich. The population shares are 65, 20 and 15 %, respectively. Two agricultural commodities are distinguished (food agriculture and cash crops) and six non-agricultural commodities (mining, fuel, local and tradable industrial products, construction and services). The rural households derive two-thirds of their income from food agriculture, and the remaining one-third from cash agriculture, industry, construction and services. Agriculture is important also for the urban poor, yielding half of their income, with the other half coming from industry, construction and services. Income of the urban rich depends for only 10 % on agriculture, whereas 35 % comes from construction and services, 30 % from mining and energy and 25 % from industry. Government revenues are largely coming from mining and energy (85 %). Exogenous transfers mainly reflect direct taxes and are rather small. The exogenous period-2 investment levels of the households are relatively low, especially for rural and urban poor. The government level is much higher, representing close to 25 % of the value of its endowments.[13]

The following uniform parameter values are used: $\delta = 0.025$, $\nu = 0.80$, whereas for σ three alternative values are explored (0.75, 1 and 1.25). Per capita commitments of the household classes are roughly based on an assumed one-quarter of the average private consumer expenditures. The remaining part of the expenditures determines the uncommitted budget shares β_{ik}. The exogenous non-stochastic endowments in period 2 are 10 % below the level of period 1. Investments must make up for this decline. Various assumptions will be made for the liquidity parameters ℓ_{it}.

As point of departure (base model) we consider a specification with $\sigma = 1$, moderate borrowing constraints (parameter ℓ_{i1} 5 % for households and 10 % for government) and unconstraining insurance parameters ℓ_{i2}. Its outcomes under certainty are presented in tables 5.1 through 5.4. Table 5.1 shows the large gap in per capita utilities and incomes between on one hand the rural population and the urban poor, and on the other hand the urban rich. Income of the latter is about twice as high.

Table 5.1 Utility and income in the base model under certainty

	Per capita utility		Per capita income	
	Period 1	Period 2	Period 1	Period 2
Rural	1.194	1.265	24.956	23.240
Urban poor	1.353	1.420	27.964	25.845
Urban rich	2.179	2.225	56.396	48.034
Government	0.520	0.492	4.081	3.807

The investment rates in table 5.2 show that the desire to invest is larger for the poor classes. The own savings of rural and urban poor are not sufficient to finance these investments and they borrow from the rich and, to a lesser extent, from government in period 1. Due to the intertemporal budget constraint, all loans have to be repaid in period 2. None of the classes is impeded by its borrowing constraint, as can be seen from the saving surplus rate. Only when

[13] These data are based on a stylized version of a Social Accounting Matrix for Nigeria, 1989, constructed at SOW-VU in the mid-nineties.

the borrowing bounds would be set at 2 % of period-1 income or less, they would become binding, at least for the poor classes. For all classes the average rate of return on investments equals 25 %, i.e. the value of $(1/\nu)-1$.

Table 5.2 Period-1 saving and investment rates in the base model under certainty

	Saving rate	Investment rate	Saving surplus rate
Rural	0.071	0.097	-0.026
Urban poor	0.073	0.095	-0.022
Urban rich	0.115	0.054	0.061
Government	0.241	0.223	0.019
Total	0.102	0.102	0.000

Table 5.3 shows that the parameters and exogenous endowment levels have been specified such that all commodity prices are lower in period 2 than in period 1, with the exception of services. Apparently, the supply of services in period 2 is relatively low, related to the depressed investment level of the urban rich. The prices allow calculation of commodity rates of interest, i.e. the extra amount of a commodity that one can obtain in period 2 by giving up one unit of the commodity in period 1. These rates of interest reflect the relative scarcities across periods and are, in the words of Fisher (1930), 'determined by the impatience to spend income and opportunity to invest it'. The aggregate interest rate, expressed in terms of the commodity basket ϑ,[14] equals 9.3 %.

Table 5.3 Commodity prices in the base model under certainty

	Period 1	Period 2
Food	1.013	0.947
Cash crops	0.697	0.650
Minerals	0.579	0.485
Fuel	1.861	1.500
Local manufactures	0.867	0.752
Tradable manufactures	3.190	2.094
Construction	0.116	0.085
Services	1.129	1.138

Table 5.4 summarizes the distribution of income and expenditures across the three population classes, for period 1 and period 2 together. The Gini coefficients take into account population sizes (neglecting inequality within each class) and are, therefore, rather low.

[14] Food, services, local manufactures and minerals are the commodities with the largest weights in the basket. The aggregate rate of interest is calculated as $r_2(\varepsilon_2) = \dfrac{p_1\vartheta}{p_2(\varepsilon_2)\vartheta} - 1$.

Table 5.4 Distribution in the base model under certainty

	Per capita total income	Per capita real total expenditure*	Per capita total utility
Rural	48.20	46.92	2.459
Urban poor	53.81	52.59	2.772
Urban rich	104.43	102.20	4.404
Gini coefficient	0.129	0.130	0.096

* in prices of period 1 using the deflator of the commodity basket ϑ

In the next sections these outcomes will be compared to situations in which the assumption of perfect foresight is abandoned. First, we introduce aggregate, widespread risk in the base model. Then, successively, alternative risk aversion parameters, subjective probability estimates and liquidity constraints are discussed.

Section 6
Simulations with widespread risk

In this section mean-preserving, widespread risk is introduced in the model. We consider two simulation runs. In one run the investment impact, i.e. the supply expansion due to investments, follows equation (2.7a) and is, therefore, subject to the same sectoral shocks as the exogenous endowments. In the other run the investment impact is assumed to be non-stochastic, following equation (2.7b).

As defined earlier, the term ‘widespread’ indicates that the risk does not remain confined to a few sectors but affects all sectors. It is simulated by independent sector-specific random shocks, drawn from uniform distributions. The shocks have zero expectation, whereas their maximal size depends on the parameter τ_n in equation (2.5). For agriculture it is assumed that the shocks may lead to an increase or decrease of 50 % compared to the ‘certainty’ level of endowments. For mining and manufacturing these percentages are assumed to be 25 %, and for construction and services 10 %. In each event (consisting of 8 random shocks) the actors are influenced differently since they have different mixtures of endowments. Hence, these runs impose considerable uncertainty on the endowments of the rural population and, to a lesser extent, on the urban poor whereas the urban rich experience the lowest endowment risk.

In the runs several mechanisms interact in smoothing the effects of uncertainty. Period-2 prices act as first line of insurance for the owners of the risky endowments, by shifting part of the risk to the consumers. Furthermore, the owners themselves react by increasing their period-1 savings (precautionary motive). The relative decrease in period-1 prices, caused by these higher savings, makes investments in period 1 more profitable leading to a new balance between savings and investments, both at a higher level than under certainty. Finally, the contingent contracts (agreed upon at the beginning of period 1) guarantee transfers of purchasing power across actors in period 2, with size and direction of the transfers depending on the outcomes of the events. These contracts have a particularly broad coverage under widespread risk, since they specify for each situation who has to pay and who will receive and how much.

Tables 6.1 – 6.6 summarize the main outcomes of the two runs specified above. For the run with deterministic investment impact, the full set of tables is added as annex C. Table 6.1 compares the saving and investment rates under uncertainty to the rates under perfect foresight. The concern for an uncertain future emerges most clearly in the run with deterministic investment impact. All actors have significantly higher saving rates than under perfect foresight. The relative increase is largest for the poor which confirms the inverse link between precautionary saving and income discussed earlier.[15] Investments go up correspondingly but with a shift towards the packages dominated by sectors with the largest risk and period-2 price increases, i.e. the packages of rural and urban poor.

Table 6.1 Effects of widespread risk on period-1 saving and investment in base model with (i) deterministic and (ii) stochastic investment impact

	Saving rate			Investment rate		
	Certainty	Uncertainty (i)	Uncertainty (ii)	Certainty	Uncertainty (i)	Uncertainty (ii)
Rural	0.071	0.083	0.075	0.097	0.117	0.103
Urban poor	0.073	0.086	0.078	0.095	0.108	0.101
Urban rich	0.115	0.125	0.121	0.054	0.051	0.053
Government	0.241	0.246	0.241	0.223	0.223	0.228
Total	0.102	0.113	0.106	0.102	0.113	0.106

In the run with stochastic investment impact the precautionary mechanism is partly offset. Apparently, investor hesitations (originating from the negative covariance between prices and volume shocks, as argued in section 3) strongly cut the role of investments in smoothing intertemporal price differences. Therefore, period-2 prices stay relatively high, and additional consumption shifts to period 1 reduce the equilibrium saving levels. In both runs the household classes remain below the borrowing bound, set at 5 % of period-1 income. The highest relative saving deficit is 3.4 %, found for rural in the run with deterministic investment impact.

Table 6.2 Effects of widespread risk on period-2 prices* in base model with (i) deterministic and (ii) stochastic investment impact

	Price	Expected price		Standard deviation	
	Certainty	Uncertainty (i)	Uncertainty (ii)	Uncertainty (i)	Uncertainty (ii)
Food	0.935	1.040	1.112	0.393	0.525
Cash crops	0.932	1.012	1.071	0.339	0.445
Minerals	0.837	0.853	0.854	0.097	0.126
Fuel	0.806	0.845	0.843	0.138	0.174
Local manufactures	0.867	0.871	0.910	0.208	0.250
Tradable manufactures	0.656	0.653	0.709	0.179	0.255
Construction	0.733	0.654	0.707	0.059	0.073
Services	1.008	1.004	1.012	0.071	0.079

*) expressed relative to the period-1 price

Table 6.2 shows the corresponding prices in period 2, expressed relative to period 1. The table confirms that, in general, relative period-2 prices are indeed higher under uncertainty,

[15] The same would hold if all uniform distributions have the same parameter τ_n and, hence, all actors would face the same relative endowment risk.

especially when the investment impact is stochastic.[16] This outcome is reflected also in the expected aggregate rate of interest which equals 6.6 % with deterministic investment impact and 3.6 % with stochastic investment impact, both lower than the 9.3 % under certainty. Furthermore, the standard deviations of the period-2 prices are considerable. For instance, whereas the maximal shock to agricultural endowments is only 50 % , the effect on the prices may easily be twice as large. Hence, a substantial part of the endowment risk is shifted to the consumer.

The effects on real expenditures and utility are presented in table 6.3. The consistent fall in expected utility, compared to perfect foresight, reflects the presence of risk aversion in the model. The table is presented basically to track possible changes in distribution across the household classes. In this respect, the Gini-coefficient of real expenditure is probably the best measure, since expected utility exaggerates the changes due to the stronger curvature of the utility function at lower income levels. The table shows that uncertainty hardly has distributional consequences. If any, the relative situation of rural and urban poor worsens somewhat due to their stronger tendency to postpone consumption to period 2 when it is more expensive.

Table 6.3 Effects of widespread risk on real* total expenditures and total utility in base model with (i) deterministic and (ii) stochastic investment impact

	Expected per capita real expenditures			Expected per capita utility		
	Certainty	Uncertainty (i)	Uncertainty (ii)	Certainty	Uncertainty (i)	Uncertainty (ii)
Rural	46.92	45.94	45.97	2.459	2.399	2.395
Urban poor	52.59	51.76	51.57	2.772	2.731	2.715
Urban rich	102.20	100.89	99.63	4.404	4.380	4.347
Gini	0.130	0.132	0.129	0.096	0.100	0.099

*) in prices of period 1 using the deflator of the commodity basket ϑ

Under uncertainty expected outcomes present only part of the picture. The probability distribution of the outcomes is equally important. Table 6.4 provides characteristics of the dispersion of per capita real expenditures. They appear to be rather symmetric around the expected level. For the poorest classes deviations may be up to 15 % and for the urban rich up to 20 %. The difference between these percentages confirms the important role of price adjustments in transmitting endowment risk to consumers. The spread in Gini-coefficient is very moderate, due to the dependence of shocks among actors and the contingent contracts. Table 6.5 provides information on the size of these contracts, expressed in terms of net insurance as defined in section 2.

[16] In sectors with relatively small random shocks an opposite effect may be found due to the fixed investment packages for each actor in the model.

Table 6.4 Measures of dispersion of per capita real expenditures under widespread risk in base model with (i) deterministic and (ii) stochastic investment impact

	Uncertainty (i)				Uncertainty (ii)			
	Expected level	Standard deviation	Minimal level	Maximal level	Expected level	Standard deviation	Minimal level	Maximal level
Rural	45.94	1.87	40.69	51.44	45.97	2.25	39.74	52.48
Urban poor	51.76	2.27	45.48	58.25	51.57	2.72	44.15	59.20
Urban rich	100.89	5.69	85.94	115.70	99.63	6.76	81.97	116.85
Gini	0.132	0.004	0.122	0.139	0.129	0.004	0.116	0.137

The size of the contingent contracts is rather small. For rural and urban poor the maximal amount does not exceed 5 % of period-1 income, with the net receipts rather symmetrically distributed around zero. For urban rich the distribution of net receipts is slightly skewed towards the positive side, with the maximal receipt about 10 % of period-1 income. Government receipts and expenditures depend on only a few sectors, making the events that it faces somewhat different from those of the household classes and its contracts relatively larger. Its maximal payment may exceed 20 % of period-1 income.

Table 6.5 Net insurance receipts in the base model under widespread risk with (i) deterministic and (ii) stochastic investment impact

	Uncertainty (i)			Uncertainty (ii)			Reference:
	Standard deviation	Minimal amount	Maximal amount	Standard deviation	Minimal amount	Maximal amount	income period 1*
Rural	17.5	-59.3	56.9	21.4	-75.6	72.3	1520
Urban poor	6.2	-19.9	20.7	7.0	-22.4	23.4	555
Urban rich	17.4	-44.5	64.7	21.2	-56.8	80.3	765
Government	19.2	-70.5	54.7	21.2	-80.8	57.4	375

*) approximation which applies to both (i) and (ii)

The limited size of the contracts is related to the aggregate character of the risk,[17] turning the Arrow-Debreu mechanism of risk sharing into a rather complicated package deal. We may obtain additional insight in these payments by calculating the correlation coefficients between net contingent receipts and nominal period-2 endowment income. They are shown in table 6.6.

[17] If the risk is not aggregate, i.e. if actors face compensating sectoral random shocks that cancel out for the sector as a whole, the size of the contingent transactions is much larger. Simulations with deterministic investment impact show that the amounts go up to one-third of period-1 income for the poor and even to 100 % of period-1 income for the rich (who are assumed to face the compensating shocks). In this case the contingent payments perfectly smooth the period-2 outcomes, making all variation in period-2 prices, utilities and expenditures disappear and the outcomes the same as under perfect foresight. It must be noted that this exact correspondence with perfect foresight is only possible since we neglect transaction costs of contingent contracts in the model.

Table 6.6 Correlations of contingent contracts under widespread risk, in the base model with (i) deterministic and (ii) stochastic investment impact

	Correlation coefficient of net contingent receipt with		
	Nominal income	Real expenditures	Utility
Uncertainty (i)			
Rural	-0.578	0.601	0.586
Urban poor	-0.038	0.074	-0.153
Urban rich	-0.013	-0.700	-0.841
Uncertainty (ii)			
Rural	-0.547	0.575	0.554
Urban poor	-0.025	0.058	-0.127
Urban rich	0.031	-0.660	-0.797

For rural households the coefficients are between –0.5 and –0.6, indicating that contingent payments smooth their income fluctuations to a considerable extent. However, for urban households the contingent payments hardly play this role, since for them the coefficients are close to zero. The table also mentions the correlation coefficients between net contingent receipts and real expenditures respectively utility. Here, the difference between the classes is even more pronounced. Rural households generally have positive receipts in situations of higher welfare and may be considered as 'overinsured', whereas for urban rich the opposite applies. For urban poor the correlations are close to zero, suggesting a rather commensurate level of insurance.

Summarizing, the simulations with widespread risk confirm the persistence of precautionary savings in this orthodox general equilibrium context, although less manifest if the investment impact is assumed to be stochastic. Price adjustments shift a considerable part of endowment risk to the consumers. The contingent contracts are rather modest and may reflect underinsurance or overinsurance, depending on the actor. Effects of risk on income distribution are only marginal.

Section 7
Alternative risk aversion parameters

As discussed in section 3, a different value of the iso-elastic parameter σ means a different curvature of the instantaneous utility function and, therefore, a different degree of risk aversion (and simultaneously a different elasticity of intertemporal substitution). So far, we considered only the value $\sigma = 1$. Here, we perform simulations with alternative values in order to see whether the conclusions of the previous section remain valid. Three values of σ will be compared, viz. 0.75, 1 and 1.50. The values $\sigma = 0.75$ and $\sigma = 1.50$ imply a degree of relative risk aversion that is one third higher respectively lower than for $\sigma = 1$. In this section the investment impact is assumed to be deterministic, whereas the liquidity constraints are as in the base model.

Table 7.1 Effects of widespread risk on period-1 saving rates, for different values of σ *

	σ = 0.75		σ =1		σ =1.50	
	Certainty	Uncertainty	Certainty	Uncertainty	Certainty	Uncertainty
Rural	0.067	0.081	0.071	0.083	0.075	0.084
Urban poor	0.067	0.083	0.073	0.086	0.079	0.089
Urban rich	0.107	0.119	0.115	0.125	0.124	0.131
Government	0.246	0.251	0.241	0.246	0.233	0.240
Total	0.098	0.111	0.102	0.113	0.106	0.115

* borrowing constraints: ℓ_{i1} = 0.05 (i<4), 0.10 (i=4); no insurance constraints; investment impact deterministic

A lower value of σ means a stronger utility curvature and, therefore, a higher degree of risk aversion. This property is reflected in table 7.1 which shows that the highest precautionary savings (i.e. the highest increases from certainty to uncertainty) are indeed found for $\sigma = 0.75$. The level of the saving rates appears to be lower for low values of σ but comparing saving rates for different values of σ says only something about the tendency towards intertemporal substitution and not about precautionary behaviour.[18]

The effects of uncertainty on relative period-2 prices are similar for the different values of σ. For $\sigma = 0.75$, the aggregate rate of commodity interest falls from 11.3 % to 9.1 %, for $\sigma = 1$ from 9.3 % to 6.6 %, and for $\sigma = 1.50$ from 7.1 % to 4.2 %. Hence, in each variant investment becomes more profitable under uncertainty. Table 7.2 shows the resulting investment rates which, for the country as a whole, must be equal to the saving rates. In all variants the household saving deficits remain well below the borrowing constraints of 5 % of period-1 income.

[18] Intertemporal price smoothing via consumption shifts is relatively strong for a high value of σ, which leads in our case (since we have a situation in which period-1 prices generally exceed period-2 prices) to an equilibrium with relatively high period-1 savings and a relatively low aggregate rate of commodity interest.

Table 7.2 Effects of widespread risk on period-1 investment rates, for different σ *

	σ = 0.75		σ =1		σ =1.50	
	Certainty	Uncertainty	Certainty	Uncertainty	Certainty	Uncertainty
Rural	0.095	0.119	0.097	0.117	0.102	0.118
Urban poor	0.084	0.099	0.095	0.108	0.100	0.109
Urban rich	0.053	0.050	0.054	0.051	0.056	0.053
Government	0.216	0.213	0.223	0.223	0.232	0.229
Total	0.098	0.111	0.102	0.113	0.106	0.115

* see note at table 7.1

The effects of uncertainty on the Gini-coefficient of per capita real total expenditures are approximately the same for different values of σ, and not reported here. The standard deviations of the period-2 prices are relatively high for low values of σ, confirming that the need for stabilization moves parallel to the degree of risk aversion. For the same reason one might expect the contingent contracts to be larger at low values of σ, but the outcomes are slightly more complicated, as table 7.3 shows.[19] The size of the contracts is modest in each variant.

Table 7.3 Spread of net insurance receipts under widespread risk, for different σ *

	Standard deviation of net insurance			Reference:
	σ = 0.75	σ =1	σ =1.50	income period 1**
Rural	17.8	17.5	19.7	1530
Urban poor	8.4	6.3	7.0	560
Urban rich	28.3	17.4	9.1	750
Government	29.8	19.2	24.4	395

* see note at table 7.1 ** approximate level applying to each of the three runs

Summarizing, the runs with alternative values for σ confirm the findings for $\sigma = 1$ about the effects of risk on the equilibrium outcomes. Only the size of the effects changes with the degree of risk aversion assumed. Precautionary saving and price variability are larger for lower levels of σ. Contingent contracts remain modest. In the next section we return to $\sigma = 1$ and study what happens with subjective probability estimates in the utility functions.

[19] If all uniform distributions have the same parameter τ_n and, hence, all actors face the same relative endowment risk, the resulting contingent contracts are unambiguously larger at lower levels of σ.

Section 8
Optimistic and pessimistic actors

So far, the expectations have been expressed in terms of density function $f(\varepsilon_2)$, which is the same for all actors and reflects the common belief of the probability of future events. In this section, actors are assumed to show a certain degree of optimism or pessimism about future events in formulating their expected utility. This optimism or pessimism is reflected in a subjective density function of the actor. The subjective density function can be applied only to expected utility and not to the expected budget constraint. Using the subjective density functions in the budget constraint would be incompatible with the assumption of market clearing in each random event in period 2, since in that case aggregation over budgets would lead to a different macro-economic balance than aggregation over commodity markets. Hence, the differences between the actors cannot be interpreted as differences in information.

Thus, we assume (i) that the actors know the common belief about the probability of future events, (ii) that they agree on the future prices in each event, and (iii) that they apply their own subjective probability estimates in ranking preferences. Assumptions (ii) and (iii) originate from Radner (1972) and are baptized the principle of ‘correct’ expectations by Magill and Quinzii (1996), as relaxation of the concept of rational expectations in which all actors must have the same probability estimates in their expected utility. Assumption (i) is not explicitly mentioned by these authors. They work with a discrete set of possible states and implicitly assign equal probabilities to each state by defining the budget constraints in terms of sums over states.[20] These equal probabilities define the commonly believed density f.

The relaxation of the concept of rational expectations is implemented in program (2.1) by replacing the common density function $f(\varepsilon_2)$ in the utility expectation by the subjective density function $f_i(\varepsilon_2|I_1)$, defined conditional upon information I_1 available in period 1. As mentioned above, the information is the same for everybody. The formulation is equivalent to multiplying the function $u_{i2}(x_{i2}(\varepsilon_2))$ by the factor $f_i^*(\varepsilon_2)$, with

$$f_i^*(\varepsilon_2) = \frac{f_i(\varepsilon_2 \mid I_1)}{f(\varepsilon_2)}\,.$$

In the simulations below, the subjective marginal density of random source n is specified as follows, for actor i:

[20] Furthermore, they allow for the possibility of incompleteness of the set of insurance contracts by imposing restrictions on states that can appear in one and the same budget constraint.

$$\frac{f_i^n(\varepsilon_{n2}\,|\,I_1)}{f^n(\varepsilon_{n2})} \begin{array}{lll} = 1+\psi_i & \text{for } 0<\varepsilon_{n2}\le\tau_n & \\ = 1 & \text{for } \varepsilon_{n2}=0 & (8.1) \\ = 1-\psi_i & \text{for } -\tau_n\le\varepsilon_{n2}<0 & \end{array}$$

The parameter $\psi_i \in (-1, 1)$ indicates the degree of optimism or pessimism of the actor. For $\psi_i > 0$ the actor is optimistic, for $\psi_i < 0$ pessimistic. Due to the independence of the N random sources, the factor $f_i^*(\varepsilon_2)$ is the product of the N outcomes of (8.1).

A pessimistic actor attaches a relatively large weight to events with reduced commodity supply and, hence, high consumer prices in period 2. Therefore, his budget constraint will force him to increase his precautionary savings. When an actor is assumed to be optimistic, the opposite effect results. However, the extent to which these partial effects will survive in the general equilibrium context, is difficult to predict. Changes in precautionary behaviour lead to changes in intertemporal price ratios and, hence, to adjustment of investment and readjustment of savings of all actors (whether pessimistic or optimistic) until the balance between saving and investment is restored. Furthermore, price variability may increase, as well as the opportunities for contingent contracts, both influencing the degree of uncertainty faced by consumers. Below, we will analyze the size of the resulting increases and decreases of savings in simulation runs in which some actors are optimistic and other pessimistic.

The simulation results are directly comparable to the results under rational expectations of section 6, in the sense that risk is widespread and the parameters are the same as in the base model. We focus on the case with deterministic investment impact. The subjective factors of (8.1) apply to each of the eight independent shocks. Two alternative specifications of correct expectations are explored. In the first one, rural and urban poor are optimistic and urban rich pessimistic. In the second one, the opposite assumptions are made. In both simulations government follows the common density function. The outcomes below are obtained with $\psi_i = 0.2$ for an optimistic actor and $\psi_i = -0.2$ for a pessimistic actor. These values seem moderate, but since the factor $1 \pm \psi_i$ is applied eight times in the joint density function, the value of $f_i^*(\varepsilon_2)$ may range from about 0.2 to 4.3.

Table 8.1 shows the effects of optimism and pessimism on saving and investment rates in the first period. In general, pessimism indeed increases saving rates, whereas optimism decreases them. However, the equilibrium changes are very moderate. Apparently, price adjustments and contingent contracts temper the initial saving reactions. For urban rich the initial saving reaction is even reversed. Due to their low population share of 15 %, the attitude of the rich appears to be dominated by the attitude of the poor. Investment goes up when the poor are pessimistic and falls when the poor are optimistic.

Table 8.1 Effects of optimism and pessimism on period-1 saving and investment rates (base model with widespread risk and deterministic investment impact)

	Rational expectations		Poor optimistic, rich pessimistic		Poor pessimistic, rich optimistic	
	Saving rate	Investm.rate	Saving rate	Investm.rate	Saving rate	Investm.rate
Rural	0.083	0.117	0.080	0.113	0.085	0.120
Urban poor	0.086	0.108	0.083	0.105	0.090	0.110
Urban rich	0.125	0.051	0.122	0.051	0.126	0.051
Government	0.246	0.223	0.245	0.220	0.247	0.225
Total	0.113	0.113	0.110	0.110	0.115	0.115

The prices in table 8.2 underline the investment reactions noted above. Compared to the run with rational expectations, expected period-2 prices are generally lower in the run with optimistic poor, and higher in the run with pessimistic poor. Furthermore, the standard deviations reveal significant increases of price variability in period 2 under subjective utility expectations, especially when the poor are pessimistic.

Table 8.2 Effects of optimism and pessimism on period-2 prices* (base model with widespread risk and deterministic investment impact)

	Rational expectations		Poor optimistic, rich pessimistic		Poor pessimistic, rich optimistic	
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.
Food	1.040	0.393	1.018	0.425	1.063	0.607
Cash crops	1.012	0.339	0.993	0.383	1.032	0.551
Minerals	0.853	0.097	0.847	0.224	0.859	0.288
Fuel	0.845	0.138	0.834	0.252	0.855	0.343
Local manuf.	0.871	0.208	0.867	0.286	0.877	0.400
Tradable manuf.	0.653	0.179	0.654	0.229	0.654	0.317
Construction	0.654	0.059	0.667	0.196	0.643	0.228
Services	1.004	0.071	1.004	0.108	1.005	0.140

* expressed relative to the period-1 price

The impact of optimism and pessimism on the model outcomes is most visible in the resulting contingent contracts which are much larger than under rational expectations. For the household classes the standard deviations are about 20 times as high, as can be seen in table 8.3. In terms of extremes, for rural poor the maximal net insurance receipt is 80 % of period-1 income, for urban poor 100 % and for urban rich even 200 %. The maximal net payments are somewhat lower, pointing to a certain skewness in the probability density of the contingent contracts.

Table 8.3 Effects of optimism and pessimism on net insurance receipts (base model with widespread risk and deterministic investment impact)

	Standard deviation of net insurance			Reference:
	Rational expectations	Poor optimistic, rich pessimistic	Poor pessimistic, rich optimistic	income period 1*
Rural	17.5	325.0	326.7	1525
Urban poor	6.3	134.1	133.2	555
Urban rich	17.4	401.4	390.7	765
Government	19.2	84.9	95.7	375

* approximate level applying to each of the three runs

Table 8.4 shows that the increased variability in period-2 outcomes is also reflected in per capita real expenditures. The figures are calculated with respect to the common beliefs about the probability of period-2 events. Under subjective utility perceptions the means are more or less the same as under rational expectations, but the standard deviations are significantly higher, especially for optimistic actors. Also the spread in the Gini-coefficient is much higher, implying that the variability differs by actor. Hence, in this case the abundant contingent contracts are sharpening the differences across classes rather than mitigating them. The expected Gini suggests a slight tendency towards more inequality.

Table 8.4 Effects of optimism and pessimism on per capita real expenditures (base model with widespread risk and deterministic investment impact)

	Rational expectations		Poor optimistic, rich pessimistic		Poor pessimistic, rich optimistic	
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.
Rural	45.94	1.87	45.42	6.76	44.50	5.10
Urban poor	51.76	2.27	51.08	7.99	50.07	5.99
Urban rich	100.89	5.69	103.29	26.93	107.16	34.89
Gini	0.132	0.004	0.141	0.074	0.149	0.082

Summarizing, the introduction of subjective utility perceptions leads to only small changes in investment and saving rates. However, it offers a great opportunity for contingent contracts across actors. These contracts appear to be much larger than under rational expectations and contribute to a considerable variability in period-2 prices and inequality. These outcomes forcefully underline the earlier finding that risk sharing in an Arrow-Debreu context is much less defensive than simply insuring oneself against adverse events.

Section 9
Liquidity constraints

The term ‘liquidity constraint’ refers to financial limitations in purchasing an asset, whether or not the asset is meant for consumption smoothing across periods or for consumption smoothing across events. For many real world assets (formal and informal, physical and financial) these two aspects are hard to disentangle, but in the theoretical setting of the illustrative Arrow-Debreu model a strict distinction can be made.[21] This distinction is reflected in the formulation of the liquidity constraints in program (2.1), indicating respectively borrowing bounds and insurance bounds. As discussed earlier, we use the term ‘insurance bounds’ in the general sense of bounds on shifting risk across actors.

So far, liquidity constraints have not really played a role in the simulations. Insurance bounds were not yet imposed, and the borrowing bounds of the base model (5 % of period-1 income for the households and 10 % of period-1 income for government) are non-binding. Although there are significant financial flows in period 1 from rich to poor, the amounts do not exceed 5 % of the income of the latter. Net borrowing by an actor reflects a relatively large desire for either consumption smoothing, due to differences in endowments between the two periods, or investment credit, due to differences in investment technologies between the actors. In the illustrative model the second explanation dominates: the poor appear to have the better investment opportunities.

In this section, we will study the effects of tightening the borrowing bounds and introducing insurance bounds. Point of departure is the simulation with widespread uncertainty and rational expectations of section 6. Again, we will focus on the case with deterministic investment impact. In the first simulation below, borrowing is excluded completely. Insurance is not yet limited.

Table 9.1 Effect on period-1 saving and investment rates of a borrowing ban, under rational expectations and widespread risk (σ and ℓ_{i2} as in base model)

	Saving rate		Investment rate	
	With borrowing	Without borrowing	With borrowing	Without borrowing
Rural	0.083	0.095	0.117	0.095
Urban poor	0.086	0.095	0.108	0.095
Urban rich	0.125	0.089	0.051	0.089
Government	0.246	0.243	0.223	0.243
Total	0.113	0.112	0.113	0.112

Table 9.1 shows that the effects of the elimination of borrowing on period-1 saving and investment patterns are significant. The rural and urban poor, previously net borrowers,

[21] As emphasized also in the introduction of Besley (1995).

increase their savings considerably whereas the urban rich, previously net lenders, cut their savings a great deal. For government the effect is similar as for urban rich, but much smaller. At the same time, the former net borrowers reduce their investments, whereas the former net lenders do the opposite. Hence, investments shift to less profitable activities. The reduced profitability of investment does not lead to a large decline of the investment rate of the whole economy (minor change from 0.113 to 0.112), but is rather reflected in significant relative price increases in period 2 compared to period 1. The aggregate rate of interest even becomes negative.

Table 9.2 Real expenditure and utility effect of a borrowing ban, under rational expectations and widespread risk (σ and ℓ_{i2} as in base model)

	Expected per capita real total expenditures		Expected per capita total utility	
	With borrowing	Without borrowing	With borrowing	Without borrowing
Rural	45.94	46.59	2.399	2.418
Urban poor	51.76	52.19	2.731	2.735
Urban rich	100.89	99.64	4.380	4.337
Gini	0.132	0.126	0.100	0.097

Measured at the welfare weights of the unconstrained run, overall expected welfare in the economy decreases due to the ban on borrowing, as it should theoretically. However, theory does not tell anything about the distribution of the loss over the actors. Table 9.2 shows that in this case there are even winners: rural and urban poor gain from the borrowing ban whereas urban rich lose. As a consequence, the Gini coefficients fall. The distribution of the losses is clearly related to the profitability of the investments of the actors. Under the borrowing ban, the urban rich cannot lend and are forced to spend more money on their own investment activities than they would prefer. Although these outcomes may easily be overturned in a different setting, they make clear that borrowing constraints do not necessarily lead to higher inequality in an intertemporal Arrow-Debreu model.

Now, we impose an insurance ban instead of a borrowing ban. In section 6 we have seen that in the base model under rational expectations and widespread risk the contingent payments are relatively small, especially for the poor. Still it comes as a surprise that saving and investment rates are hardly or not affected by the ban on insurance, as table 9.3 shows. The same applies to the prices and their standard deviations. The absence of increases in precautionary saving is a signal that, in the perception of the actors, the limitations on the contingent contracts do not significantly raise the degree of risk that they face. This conclusion is confirmed by the overall expected welfare loss which appears to be very small, much smaller than under the borrowing ban.

Table 9.3 Effect on period-1 saving and investment rates of an insurance ban*, under rational expectations and widespread risk (σ and ℓ_{i1} as in base model)

	Saving rate		Investment rate	
	With insurance	Without insurance	With insurance	Without insurance
Rural	0.083	0.082	0.117	0.116
Urban poor	0.086	0.085	0.108	0.108
Urban rich	0.125	0.124	0.051	0.052
Government	0.246	0.248	0.223	0.221
Total	0.113	0.113	0.113	0.113

*) in fact, not a complete ban but insurance maximally 0.5 % of period-1 income

Table 9.4 shows that also the distribution of expected real expenditures hardly changes. The only difference across the classes refers to the standard deviation of real expenditures. It increases slightly for urban rich, remains unchanged for urban poor and decreases slightly for rural. These movements reflect the different character of the contingency payments for the different classes. For classes with underinsurance (positive receipts coinciding with lower real expenditures) the standard deviation increases when the payments are bounded, for classes with overinsurance the opposite happens. The standard deviation of the Gini increases somewhat, as one would expect when the possibility of risk sharing is reduced.[22]

Table 9.4 Effect on real expenditures of an insurance ban*, under rational expectations and widespread risk (σ and ℓ_{i1} as in base model)

	Expected per capita real total expenditures		Standard deviation of per capita real total expenditures	
	With insurance	Without insurance	With insurance	Without insurance
Rural	45.94	45.95	1.87	1.77
Urban poor	51.76	51.79	2.27	2.26
Urban rich	100.89	101.06	5.69	6.40
Gini	0.132	0.132	0.004	0.006

*) in fact, not a complete ban but insurance maximally 0.5 % of period-1 income

The insignificance of the saving effects of the insurance bounds may be due to the relatively small size of the contingent payments under rational expectations. Under subjective expected utility perceptions the contingent transactions are much larger, as seen in section 8. Therefore, we explore the effects of insurance bounds also when actors are optimistic or pessimistic. In this case, we consider bounds on net insurance receipts that are equal to 10 % of period-1 income for all actors. These bounds are substantial limitations, given the size of the unconstrained receipts which may amount to 100 % of period-1 income for the poor and even 200 % for the rich. Apart from these bounds, the specification of the simulations is the

[22] If in the simulation above τ_n is set at 0.4 for all n, hence if the actors face the same relative endowment risk, all household classes appear to be overinsured in terms of real expenditures. Then, the insurance ban leads to a lower standard deviation of real expenditures of all household classes. However, the change in the standard deviation of the Gini still increases.

same as in section 8. Again, we distinguish a specification with the poor optimistic and the rich pessimistic, and a specification with the poor pessimistic and the rich optimistic.

The bounds on insurance lead to a decline in expected utility (subjectively measured) for all actors, with the largest decline faced by the urban rich. Likewise, their insurance constraint has by far the largest expected shadow price in terms of aggregate welfare, $\breve{\mu}_{i2}(\varepsilon_2)$, irrespective of who is assumed to be optimistic or pessimistic. Hence, especially the rich are affected by the constraints on insurance. The changes in saving rates are related to the relative importance of the contingent contracts, rather than to the assumptions of optimism and pessimism, as shown in table 9.5. The saving rates of the rich increase considerably in both runs, whereas the rates of the poor fall. Apparently, the rich feel much more insecure when contingent payments are limited.

Table 9.5 Effect on period-1 saving rates of insurance bounds*, under widespread risk and with optimistic and pessimistic actors (σ and ℓ_{i1} as in base model)

	Poor optimistic, rich pessimistic		Poor pessimistic, rich optimistic	
	Insurance free	Insurance bounded	Insurance free	Insurance bounded
Rural	0.080	0.071	0.085	0.073
Urban poor	0.083	0.074	0.090	0.075
Urban rich	0.122	0.150	0.126	0.158
Government	0.245	0.253	0.247	0.263
Total	0.110	0.112	0.115	0.117

*) net insurance receipts maximally 10 % of period-1 income

Since the urban rich suffer most from the bounds, there is a tendency towards more equality. The expected Gini coefficient of real expenditures falls from 0.141 to 0.137, respectively from 0.149 to 0.141. Furthermore, since in the unconstrained simulations of section 8 all actors are overinsured, whether optimistic or pessimistic, the insurance bounds lead to uniformly lower standard deviations of the real expenditures. This time even the fluctuations in the Gini, quite significant in the unconstrained simulations, are reduced by the insurance bounds.

Summarizing, under a borrowing ban we find significant saving reactions and welfare losses, since the actors with the less profitable investment opportunities are forced to invest more than they actually prefer. Under an insurance ban, saving reactions and welfare losses are much smaller, at least under rational expectations. Only when the assumption of rational expectations is abandoned, saving reactions to insurance bounds are considerable, especially for actors with relatively large contingent contracts.

Section 10
Summary of findings

In this paper we have studied the effect of aggregate risk on saving and investment in an illustrative two-period general equilibrium model with contingent contracts (possibly constrained) in the Arrow-Debreu tradition and rather standard functional forms. Risk is modelled in the form of independent sector-specific shocks. All contracts are respected and the model is closed. Actors maximize expected utility. In a partial setting this specification of preferences and behaviour would imply that risk leads to precautionary saving, with the largest tendency for the poorest classes. In the general equilibrium model this conclusion cannot be drawn a priori since prices and investments adjust and contingent contracts are possible. The level of an investment activity depends on its profitability and is, therefore, directly related to the endogenous prices.

Simulations with the two-period model show that precautionary saving persists also in a general equilibrium context, in spite of the equilibrium price changes and the smoothing provided by contingent contracts. Aggregate risk may explain an increase of the national period-1 saving rate from 0.10 to 0.12, hence a relative increase of 20 %. For poor classes the differences are more pronounced. Under uncertainty, their period-1 savings may be 25 % higher. However, the precautionary saving effect is drastically lower if investment itself is stochastic, more precisely if its impact is positively correlated to the aggregate risk in the economy.

As expected, precautionary savings appear to be larger if the degree of risk aversion is higher, or if pessimism is the dominating attitude in the economy. Surprisingly, savings hardly react to constraints on contingent transactions. Only when the assumption of rational expectations is dropped and household classes are assumed to be optimistic or pessimistic in the perception of their expected utility, bounds on contingent transactions have a significant impact on saving rates. Then, savings shift towards the classes for which the contingent contracts are relatively most important (in our illustrative model, the rich).

With respect to the Arrow-Debreu contingent contracts, the simulations show that under aggregate risk their objective goes beyond providing insurance against adverse events. The contracts also provide opportunities in case of good events. Households turn out to be overinsured or underinsured, in the sense of having a positive respectively negative correlation between net contingent receipts and real expenditures.

Finally, we observe that aggregate risk hardly affects the expected distribution of expenditures across household classes, at least without liquidity bounds. Borrowing bounds prove to harm especially the households with ‘bad’ investment activities, whether rich or poor. Insurance bounds appear to lead to less inequality, but it is not immediately clear how specific this finding is for the current model. Anyhow, the simulations show that in an Arrow-Debreu context liquidity constraints are not necessarily detrimental to the poor.

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Annex A
Model symbols

Indices:

i	actors (1,…,m)
k	commodities (1,…,r)
n	random sources (1,…,N)
t	periods (1,…,T)

Variables:

d_t	m-vector of investment levels in period t
d_{it}	investment level of actor i in period t (scalar)
$\bar{d}_{iT}$	exogenous investment level of actor i in period T
h_{it}	income of actor i in period t
$\bar{n}_i$	population size of actor i (scalar, exogenous)
p_t	r-vector of commodity prices in period t
$\tilde{p}_t$	r-vector of price parameters in period t
r_t	aggregate commodity rate of interest in period t
$\bar{t}_{it}$	net exogenous transfer receipts of actor i in period t (scalar, expressed in baskets), with $\sum_i \bar{t}_{it} = 0$
x_t	m x r matrix of consumption in period t
x_{it}	r-vector of consumption of actor i in period t
V_{it}	budget deficit of actor i in period t
Z_{it}	upper bound on borrowing (t=1) or insurance (t=2) for actor i in period t
α_i	welfare weight of actor i (scalar)
λ_i	shadow price of intertemporal budget constraint of actor i (scalar)
μ_{it}	shadow price (in terms of utility) of liquidity constraint of actor i in period t ($t < T$)
$\breve{\mu}_{it}$	shadow price (in terms of welfare) of liquidity constraint of actor i in period t ($t < T$)
ω_{it}	r-vector of endowments of actor i in period t ($t > 1$)
$\bar{\omega}_{i1}$	r-vector of exogenous endowments of actor i in period 1
$\bar{\omega}^{\circ}_{it}$	r-vector of exogenous non-stochastic endowments of actor i in period t ($t > 1$)

Functional forms:

f	density function of the random event of period 2
f_i	actor i's subjective density function of the random event of period 2
f_i^*	ratio of actor i's subjective density f_i to common density f
f^n	marginal density function of random source n of period 2
f_i^n	actor i's subjective marginal density function of random source n of period 2
g_i	vector-function (dimension r) of investment-induced supply expansion of actor i
u_{it}	instantaneous utility function of actor i in period t
v	iso-elastic transformation function
w_i	Stone-Geary function of actor i
$\overline{\omega}_{it}$	vector-function (dimension r) of exogenous endowments of actor i in period t (t > 1)

Coefficients:

ℓ_{it}	coefficient in liquidity constraint of actor i in period t
β_{ik}	budget share of commodity k in uncommitted consumption of actor i, within each period
δ	time preference parameter
γ_{ik}	committed level of consumption of commodity k by actor I
η_i	r-vector of commodity demand per unit of investment of actor i
ν	elasticity of supply expansion with respect to investment
ϑ	r-vector with composition of commodity basket (to express transfers)
σ	parameter of the iso-elastic utility transformation
τ_n	parameter of uniform density function of random source n
ψ_i	degree of optimism or pessimism in subjective density function
ζ_i	reference rate (at $d_{i1} = 1$) of investment-induced supply expansion of actor i

Other symbols:

I_1	information available in period 1
ε_2	random event (N-vector of random sources) in period 2
$\iota(k)$	pointer of sector k to one of the random sources
s	very small positive value

Annex B
Precautionary savings and income level

Here, we study the relation between precautionary savings and level of income in a partial context, for an intertemporally additive, iso-elastic specification of utility. We consider an actor i who maximizes expected two-period utility subject to intertemporal budget constraints. Income in both periods is exogenous. Period-2 income is stochastic and unknown in period 1. Utility is derived from consumption aggregates X_{it}, of which prices P_t are given to the actor. Only income uncertainty is considered, hence prices are non-stochastic. Budget constraints are imposed for each possible realization of income in period 2. Insurance is not considered as decision variable. Its effects are assumed to be included already in the stochastic income specification.

The instantaneous utility functions are given by u_{it} and uncertainty is represented by a scalar ε with density function $f_i(\varepsilon)$. Income is specified as function y_i with arguments $\bar{h}_i$ and ε, of which the former represents expected income, also equal to income under certainty. Hence, $\int y_i(\bar{h}_i,\varepsilon) f_i(\varepsilon)\, d\varepsilon = \bar{h}_i$ and $y_i(\bar{h}_i, 0) = \bar{h}_i$. The actor's maximization problem, with shadow prices between brackets:

$$\max_{X_{i1}, X_{i2}(\varepsilon)\geq 0} \quad u_{i1}(X_{i1}) + \int u_{i2}(X_{i2}(\varepsilon))\, f_i(\varepsilon)\, d\varepsilon$$

$$\text{s.t.} \quad P_1 X_{i1} + P_2 X_{i2}(\varepsilon) \leq y_i(\bar{h}_i, \varepsilon) \qquad (\lambda_i(\varepsilon)) \qquad \text{for all } \varepsilon$$

$$\text{with} \quad u_{it}(X_{it};\sigma) = \begin{cases} e^{-\delta(t-1)}\,(X_{it}^{\,1-1/\sigma} - 1)/(1-1/\sigma) & \text{for } \sigma > 0, \sigma \neq 1 \\ e^{-\delta(t-1)} \log(X_{it}) & \text{for } \sigma = 1 \end{cases}$$

With this specification, income is fully spent and consumption is strictly positive in each period and event. Therefore, first-order conditions can be written as:

$$P_2 \frac{\partial u_{i1}}{\partial X_{i1}} = P_1 \int \frac{\partial u_{i2}}{\partial X_{i2}(\varepsilon)} f_i(\varepsilon)\, d\varepsilon \qquad \text{(b.1)}$$

$$P_2 X_{i2}(\varepsilon) = y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1} \qquad \text{(b.2)}$$

with $$\frac{\partial u_{it}}{\partial X_{it}} = e^{-\delta(t-1)}\ X_{it}{}^{-1/\sigma} \qquad \text{for } \sigma > 0 \tag{b.3}$$

Substitution of (b.2) and (b.3) in (b.1) gives

$$\int [P_1 X_{i1} /(y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1})]^{1/\sigma} f_i(\varepsilon)\, d\varepsilon = e^{\delta}[P_1 / P_2]^{-1+1/\sigma} \tag{b.4}$$

To analyze this condition, we define

$$D_i(\bar{h}_i; X_{i1}) = \int \chi_i(\varepsilon; X_{i1}, \bar{h}_i)\, f_i(\varepsilon)\, d\varepsilon \; - \; \chi_i(0; X_{i1}, \bar{h}_i)$$

with $\chi_i(\varepsilon; X_{i1}, \bar{h}_i) = [P_1 X_{i1} /(y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1})]^{1/\sigma}$

The function $D_i(\bar{h}_i; X_{i1})$ measures the increase in the lefthandside of (b.4) under the introduction of income uncertainty, at given X_{i1}. Therefore, it is also a measure of the relative decrease in X_{i1} that is necessary to restore equality with the righthandside, which remains unaltered at given prices P_1 and P_2. If we can derive

a) $D_i(\bar{h}_i; X_{i1}) \geq 0$

b) $\dfrac{\partial D_i(\bar{h}_i; X_{i1})}{\partial \bar{h}_i} \leq 0$

then we know that the precautionary saving effect indeed exists (from a) and that the effect decreases when income increases (from b).

First we consider (a). Differentiation of χ_i with respect to ε gives:

$$\frac{\partial \chi_i(\varepsilon; X_{i1}, \bar{h}_i)}{\partial \varepsilon} = -\frac{1}{\sigma}(P_1 X_{i1})^{1/\sigma}(y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1})^{-(1+1/\sigma)}\left(\frac{\partial y_i}{\partial \varepsilon}\right)$$

$$\frac{\partial^2 \chi_i(\varepsilon; X_{i1}, \bar{h}_i)}{\partial \varepsilon^2} = \frac{1}{\sigma}\left(1+\frac{1}{\sigma}\right)(P_1 X_{i1})^{1/\sigma}(y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1})^{-(2+1/\sigma)}\left(\frac{\partial y_i}{\partial \varepsilon}\right)^2 -$$

$$\frac{1}{\sigma}(P_1 X_{i1})^{1/\sigma}(y_i(\bar{h}_i, \varepsilon) - P_1 X_{i1})^{-(1+1/\sigma)}\left(\frac{\partial^2 y_i}{\partial \varepsilon^2}\right)$$

If $\partial^2 y_i / \partial \varepsilon^2 \leq 0$, then χ_i is convex in ε, and Jensen's inequality implies $D_i(\bar{h}_i; X_{i1}) \geq 0$.

Then we consider (b). Assuming that we can differentiate D_i with respect to $\overline{h}_{i1}$ by differentiating the function under the integral,[23] we obtain:

$$\frac{\partial D_i(\overline{h}_i; X_{i1})}{\partial \overline{h}_i} = -\int \varphi_i(\varepsilon; X_{i1}, \overline{h}_i)(\partial y_i / \partial \overline{h}_i) f_i(\varepsilon)\, d\varepsilon \;+\; \varphi_i(0; X_{i1}, \overline{h}_i)$$

with $\quad \varphi_i(\varepsilon; X_{i1}, \overline{h}_i) = (1/\sigma)(P_1 X_{i1})^{1/\sigma}(y_i(\overline{h}_i, \varepsilon) - P_1 X_{i1})^{-(1+1/\sigma)}$

Convexity of φ_i can be derived similarly as for χ_i , via

$$\frac{\partial \varphi_i(\varepsilon; X_{i1}, \overline{h}_i)}{\partial \varepsilon} = -\frac{1}{\sigma}(1+\frac{1}{\sigma})(P_1 X_{i1})^{1/\sigma}(y_i(\overline{h}_i, \varepsilon) - P_1 X_{i1})^{-(2+1/\sigma)}(\frac{\partial y_i}{\partial \varepsilon})$$

$$\frac{\partial^2 \varphi_i(\varepsilon; X_{i1}, \overline{h}_i)}{\partial \varepsilon^2} = \frac{1}{\sigma}(1+\frac{1}{\sigma})(2+\frac{1}{\sigma})(P_1 X_{i1})^{1/\sigma}(y_i(\overline{h}_i, \varepsilon) - P_1 X_{i1})^{-(3+1/\sigma)}(\frac{\partial y_i}{\partial \varepsilon})^2 -$$

$$\frac{1}{\sigma}(1+\frac{1}{\sigma})(P_1 X_{i1})^{1/\sigma}(y_i(\overline{h}_i, \varepsilon) - P_1 X_{i1})^{-(2+1/\sigma)}(\frac{\partial^2 y_i}{\partial \varepsilon^2})$$

Hence, we find, again under the condition that $\partial^2 y_i / \partial \varepsilon^2 \le 0$, that φ_i is convex in ε. Assuming further that $\partial y_i / \partial \overline{h}_i = 1$, Jensen's inequality shows that $\frac{\partial D_i(\overline{h}_i; X_{i1})}{\partial \overline{h}_i} \le 0$ which means that also (b) holds.

Finally, a remark about the function $y_i(\overline{h}_i, \varepsilon)$. In the derivation above, four conditions were imposed on this function:

$$\int y_i(\overline{h}_i, \varepsilon) f_i(\varepsilon)\, d\varepsilon = \overline{h}_i \;,\; y_i(\overline{h}_i, 0) = \overline{h}_i \,,\; \partial^2 y_i / \partial \varepsilon^2 \le 0 \text{ and } \partial y_i / \partial \overline{h}_i = 1.$$

A linear formulation as $y_i(\overline{h}_i, \varepsilon) = (\overline{h}_i - \overline{h}_{i2}) + \overline{h}_{i2}(1+\varepsilon)$, with $\overline{h}_{i2}$ interpreted as expected period-2 income and $(\overline{h}_i - \overline{h}_{i2})$ as period-1 income, fulfils these conditions.

[23] This assumption seems justified for this specification of χ_i and for density functions of the usual type. See e.g. Apostol (1969), theorem 9.37 (proper integral) and theorems 14.23 and 14.24 (improper integral).

Annex C
Outcomes of the base model under widespread risk

10/12/01 11:10:05

List of Tables

=== "Output simulation run BLIFRE2.ANA " ===

```
::::::::::::::::::::::::::::::::::::::::::::::::::::
::                                                ::
::        Commodity balances, mean values         ::
::                                                ::
::::::::::::::::::::::::::::::::::::::::::::::::::::
```

	PERIOD1	PERIOD2
== food agriculture ==		
Endowment volume	1374.86	1438.21
Consumption volume	1374.86	1438.21
Supply surplus	0.00	0.00
== cash agriculture ==		
Endowment volume	163.10	171.85
Consumption volume	163.10	171.85
== mining ==		
Endowment volume	334.73	387.75
Consumption volume	334.73	387.75
Supply surplus	0.00	0.00
== fuel ==		
Endowment volume	84.08	94.83
Consumption volume	84.08	94.83
Supply surplus	0.00	0.00
== local industrial goods ==		
Endowment volume	359.38	364.10
Consumption volume	231.16	263.28
Investment volume	128.22	100.83
Supply surplus	0.00	0.00
== tradable industrial goods ==		
Endowment volume	187.03	208.53
Consumption volume	118.17	154.29
Investment volume	68.85	54.25
Supply surplus	0.00	0.00
== construction ==		
Endowment volume	232.43	237.81
Consumption volume	92.87	137.90
Investment volume	139.56	99.91
Supply surplus	0.00	0.00
== services ==		
Endowment volume	364.36	357.83
Consumption volume	364.36	357.83
Supply surplus	0.00	0.00

Commodity prices

	PERIOD1	PERIOD2
== Mean value ==		
food agriculture	0.998	1.038
cash agriculture	0.687	0.696
mining	0.573	0.489
fuel	1.834	1.549
local industrial goods	0.892	0.777
tradable industrial goods	3.364	2.196
construction	0.128	0.084
services	1.134	1.139
== Standard deviation ==		
food agriculture	0.000	0.392
cash agriculture	0.000	0.233
mining	0.000	0.056
fuel	0.000	0.252
local industrial goods	0.000	0.186
tradable industrial goods	0.000	0.604
construction	0.000	0.008
services	0.000	0.081
== Mean value scaled to period1 ==		
food agriculture	1.000	1.040
cash agriculture	1.000	1.012
mining	1.000	0.853
fuel	1.000	0.845
local industrial goods	1.000	0.871
tradable industrial goods	1.000	0.653
construction	1.000	0.654
services	1.000	1.004
== Std.dev. scaled to price period1 ==		
food agriculture	0.000	0.393
cash agriculture	0.000	0.339
mining	0.000	0.097
fuel	0.000	0.138
local industrial goods	0.000	0.208
tradable industrial goods	0.000	0.179
construction	0.000	0.059
services	0.000	0.071
== Expected interest rate of basket ==		
Total commodities	0.066	0.000

Income accounts, mean values

	PERIOD1	PERIOD2
== Income from endowments ==		
rural population	1523.81	1441.18
urban poor population	555.05	515.76
urban rich population	767.49	640.85
government	376.36	347.23
Total actors	3222.72	2945.03
== Income from transfers ==		
rural population	1.65	-0.38
urban poor population	1.31	-0.68
urban rich population	-17.78	-12.63
government	14.82	13.69
Total actors	0.00	0.00
== Value of consumption ==		
rural population	1399.55	1331.91
urban poor population	508.51	485.27
urban rich population	655.99	632.67
government	294.80	289.34
Total actors	2858.85	2739.19
== Value of investment ==		
rural population	178.28	56.33
urban poor population	60.03	17.38
urban rich population	38.54	50.03
government	87.02	82.10
Total actors	363.87	205.84
== Income surplus ==		
rural population	-52.37	52.56
urban poor population	-12.18	12.44
urban rich population	55.18	-54.48
government	9.36	-10.52
Total actors	0.00	0.00

Utility and income

	PERIOD1	PERIOD2	TOTAL
== Population ==			
rural population	61.100	61.100	61.100
urban poor population	19.740	19.740	19.740
urban rich population	13.160	13.160	13.160
Total actors	94.000	94.000	94.000
== Per caput utility ==			
rural population	1.174	1.226	2.399
urban poor population	1.342	1.389	2.731
urban rich population	2.177	2.204	4.380
government	0.531	0.507	1.039
== Per caput income ==			
rural population	24.967	23.581	48.548
urban poor population	28.185	26.093	54.278
urban rich population	56.969	47.737	104.706
government	4.161	3.840	8.001

Saving, investment and liquidity

	RURAL	URBANPOOR	URBANRICH	GOVERNMENT
== Rates period 1 ==				
Rate of saving	0.0825	0.0860	0.1250	0.2464
Rate of investment	0.1169	0.1079	0.0514	0.2225
Rate of saving surplus	-0.0343	-0.0219	0.0736	0.0239
== Return on investment ==				
Rate excluding shadow prices	0.2480	0.2485	0.2486	0.2462
Rate including shadow prices	0.2480	0.2485	0.2486	0.2462
== Shadow prices of constraints ==				

Distribution and insurance

	MEANVALUE	STANDDEV	MINIMUM	MAXIMUM
== Per caput nominal total income ==				
rural population	48.548	1.661	45.944	53.967
urban poor population	54.278	1.490	51.461	59.812
urban rich population	104.706	1.207	101.720	109.456
Gini coefficient	0.128	0.007	0.108	0.141
== Per caput real total expenditure ==				
rural population	45.940	1.868	40.688	51.443
urban poor population	51.763	2.268	45.483	58.251
urban rich population	100.893	5.689	85.944	115.697
Gini coefficient	0.132	0.004	0.122	0.139
== Per caput total utility ==				
rural population	2.399	0.194	1.880	2.771
urban poor population	2.731	0.194	2.212	3.103
urban rich population	4.380	0.194	3.861	4.752
Gini coefficient	0.100	0.007	0.088	0.123
== Net insurance receipt ==				
rural population	0.000	17.533	-59.295	56.858
urban poor population	0.000	6.249	-19.919	20.680
urban rich population	0.000	17.423	-44.510	64.652
government	0.000	19.232	-70.542	54.678

Insurance correlations

	RURAL	URBANPOOR	URBANRICH	GOVERNMENT
Correlation with nom. income	-0.578	-0.038	-0.013	-0.874
Correlation with real expend.	0.601	0.074	-0.700	0.194
Correlation with utility	0.586	-0.153	-0.841	-0.302

MODEL SPECIFICATION AND CONVERGENCE REPORT: BLIFRE2.ANA

Model option: Rational expectations

Number of random sources: 8   Investment function not stochastic

Parameters tau and correlation matrix:

	EVENT1	EVENT2	EVENT3	EVENT4	EVENT5	EVENT6	EVENT7	EVENT8
Tau	.50	.50	.25	.25	.25	.25	.10	.10
FOODAGRIC	1	0	0	0	0	0	0	0
CASHAGRIC	0	1	0	0	0	0	0	0
MINING	0	0	1	0	0	0	0	0
FUEL	0	0	0	1	0	0	0	0
LOCMANUF	0	0	0	0	1	0	0	0
IMPMANUF	0	0	0	0	0	1	0	0
CONSTRUCT	0	0	0	0	0	0	1	0
SERVICES	0	0	0	0	0	0	0	1

Parameters:	Sigma	Liquidity-1	Liquidity-2
RURAL	1.000	.050	25.000
URBANLOW	1.000	.050	25.000
URBANHIGH	1.000	.050	25.000
GOVERNMENT	1.000	.100	25.000

Results from welfare iteration 10000
*** Warning: MAXITW reached

Expected welfare   20796.832
Investment levels D(I,1)

INVEST	RURAL	2.282
INVEST	URBANLOW	2.493
INVEST	URBANHIGH	.557
INVEST	GOVERNMENT	.765

Test on first-order conditions of period one in welfare iteration 10000

Equality of marginal consumption costs and returns:

		RURAL	URBANLOW	URBANHIGH	GOVERNMENT
FOODAGRIC	Costs	.998	.998	.998	.998
FOODAGRIC	Returns	.998	.998	.998	.000
CASHAGRIC	Costs	.687	.687	.687	.687
CASHAGRIC	Returns	.687	.687	.687	.000
MINING	Costs	.573	.573	.573	.573
MINING	Returns	.573	.573	.573	.573
FUEL	Costs	1.834	1.834	1.834	1.834
FUEL	Returns	1.834	1.834	1.834	.000
LOCMANUF	Costs	.892	.892	.892	.892
LOCMANUF	Returns	.892	.892	.892	.000
IMPMANUF	Costs	3.364	3.364	3.364	3.364
IMPMANUF	Returns	3.364	3.364	3.364	.000

CONSTRUCT	Costs	.128	.128	.128	.128
CONSTRUCT	Returns	.128	.128	.128	.000
SERVICES	Costs	1.134	1.134	1.134	1.134
SERVICES	Returns	1.134	1.134	1.134	1.134

Equality of marginal investment costs and returns:

RURAL	78.106	77.981
URBANLOW	24.082	24.052
URBANHIGH	69.248	69.172
GOVERNMENT	113.785	113.442

Complementarity between excess supply and prices:

FOODAGRIC	.000	.998
CASHAGRIC	.000	.687
MINING	.000	.573
FUEL	.000	1.834
LOCMANUF	.000	.892
IMPMANUF	.000	3.364
CONSTRUCT	-.001	.128
SERVICES	.000	1.134

Complementarity between excess liquidity and shadow prices:

RURAL	23.869	.000
URBANLOW	15.635	.000
URBANHIGH	92.668	.000
GOVERNMENT	48.536	.000

Summary welfare iteration 10000

Actor	Alpha	Budget gap	Relative gap	Totsupval	Inv 1
RURAL	953.6957	-.1983	-.0001	2964.9969	2.282
URBANLOW	364.4653	-.2618	-.0002	1070.8155	2.493
URBANHIGH	559.9607	-.6980	-.0005	1408.3362	.557
GOVERNMENT	231.2040	1.1579	.0016	723.5938	.765

Actor	Liqbound 1	Last change	Liqbound 2	Last change
RURAL	76.2765	-.0015	38085.7643	-.2420
URBANLOW	27.8203	-.0007	13897.7701	-.0752
URBANHIGH	37.4881	-.0009	18798.8494	-.0335
GOVERNMENT	39.1231	-.0021	9790.1807	-.1391

Evaluation of random drawings:

Source	Drawings	Average	Variance
EVENT1	2042922	-.00037	.08327
EVENT2	2042922	-.00013	.08339
EVENT3	2042922	-.00003	.02082
EVENT4	2042922	.00000	.02082
EVENT5	2042922	-.00010	.02084
EVENT6	2042922	-.00008	.02082
EVENT7	2042922	.00002	.00333
EVENT8	2042922	.00001	.00333

Algorithmic summary:

Loop	Calls	Iterations per call
WELFARE	1	10000
PERIOD1-SQG	10000	204
PERIOD2	2042922	1

The Centre for World Food Studies (Dutch acronym SOW-VU) is a research institute related to the Department of Economics and Econometrics of the Vrije Universiteit Amsterdam. It was established in 1977 and engages in quantitative analyses to support national and international policy formulation in the areas of food, agriculture and development cooperation.

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