



Stichting Onderzoek Wereldvoedselvoorziening van de Vrije Universiteit

Centre for World Food Studies

**REGULATION AND EFFICIENCY IN THE HEALTH SECTOR:
A general equilibrium analysis with endogenous risk**

by

M.A. Keyzer

Content

Abstract	v
1. Introduction	1
<i>Overview</i>	2
2. Model specification	3
<i>Prevalence of illness</i>	3
<i>Consumer-patients</i>	4
<i>Endogenous probabilities: prevention and access to treatment</i>	7
<i>Production</i>	9
<i>Market equilibrium and feasibility</i>	11
3. Equilibrium: existence, Pareto efficiency and other properties	12
4. Extending the model	19
<i>Disaster</i>	19
<i>Uncertainty about the effectiveness of treatment</i>	20
<i>Consumption effects on prevention (nutrition)</i>	21
<i>Treatment in successive periods: aggregate risk</i>	21
<i>Towards a technical formulation</i>	24
5. Health policies	26
<i>Implementing the equilibrium</i>	26
<i>Prohibition</i>	27
<i>Subsidies</i>	28
<i>Equal access restrictions</i>	28
6. Conclusion	29
References	31

Abstract

Public authorities tend to be actively involved in the health sector through prevention programs, regulation of health insurance and control of access to treatment. Common explanations include asymmetries in information, the danger of epidemics and imperfect competition. This paper¹ argues that these interventions may also be required to deal with the typical externality that probabilities depend on individual and collective decisions. We formulate a general equilibrium model, in which individuals face endogenous probabilities of incurring specified diseases and obtaining timely treatment. Health insurance organizations collect premiums and invest in treatment capacity, in accordance with the preferences of their customers, but because of limited treatment capacity and insurability, they may have to conduct lotteries to assign patients. The externality causes a non-convexity in the patient's expected utility function. We show that the associated consumer demand functions are well behaved nonetheless, and that an equilibrium exists. Moreover, if bounds on insurability are not effective and the collective decisions on prevention follow Lindahl pricing, this equilibrium will be Pareto-efficient in terms of expected utility, despite the capacity constraints and the endogenous risk. We also discuss how subsidies, prohibition and equal access restrictions can be included, at the expense of efficiency.

¹ The paper benefited from comments by Geert Overbosch, T.N. Srinivasan and Lia van Wesenbeeck.

1. Introduction

Combining efficiency of service provision with equity of access has become a critical challenge to health policies worldwide. In developed countries, the aging of the population and the advancement of technological know how exacerbate classic dilemmas on how to divide the efforts between prevention and treatment, how much to spend on various patient categories, and whether doctors may give priority to patients who pay more. In developing countries, urbanization, the transition from traditional to high tech medicine, the emergence of HIV, and the scarcity of public funds give rise to even more tantalizing problems of selection of affordable treatments.

Many of these issues are extensively studied in health economics (see e.g., Zweifel and Breyer, 1997, Jack, 1999). This literature highlights the specificity of the field by pointing to typical market imperfections such as the high setup costs of facilities, the R&D intensity of the pharmaceuticals, the public good aspects of health, the uncertainty of diagnosis, and the asymmetric information between the patient and the insurer on the one hand, and between the patient and the doctor on the other. Consequently, health economics currently separates into a diversity of subjects, with contract theory, partial equilibrium analysis and industrial organization as main tools of analysis. In this field welfare theory is applied to compare very specific policy alternatives in the sphere of, say, pricing of drugs, rather than to formulate sector wide policies. In a recent survey chapter of the Handbook of Health Economics, Hurley (2000) elaborates on the many objections raised in this domain against the application of welfare economics as a normative framework. Reference is made to the above mentioned market imperfections as well as to the moral argument that life is priceless. It is concluded that a pluralist approach is called for that accepts some degree of regulation. Indeed, the common practice distinguishes many regulatory mechanisms, such as the prescription by the doctor, the admission procedure to a clinic, the approval of treatments by the health insurer, and the government regulation to avoid exclusion and maintain equal access. Such controls act as rationing schemes that may, according to welfare analysis hamper the sector's efficiency. And yet, sector specialists generally recognize their merits as coordination devices.

In this paper, we use welfare analysis as a modeling tool rather than as a prescription. This enables us to include egalitarian requirements with respect to the access to health facilities, thus highlighting the tradeoffs between equity and efficiency. It appears that under an appropriate implementation, the above mentioned regulatory mechanisms can become the very instruments for achieving Pareto-efficient outcomes.

The analysis relies on general as opposed to partial equilibrium, to account for the fact that the patient's budget constraint matters, in developed largely in relation to the cost of treatment and in developing countries also because poverty is associated with

malnutrition and poor hygienic conditions that are basic causes of illness itself. Surprisingly, and possibly because of the objections raised against welfare theory, applied general equilibrium (AGE-) models that distinguish a health sector only incorporate of separate commodities with associated supply and demand but to our knowledge there is no applied general equilibrium model with an explicit representation of the natural environment, patients, insurers and providers of health services and drugs. Our paper is intended as a preparatory step towards AGE-modeling.

Compared to the Arrow Debreu model with contingent markets (1959), the proposed model has distinctive features in its treatment of uncertainty. First, it has endogenous risk, as it deals with the probabilities of the consumer's expected utility maximization as endogenous signals rather than as constants, and generates adjustments in these probabilities through prevention and treatment. Second, besides supply and demand of commodities, it represents the access to health care as the assignment of individual consumers to discrete treatments, as in Dasgupta and Ray (1986) but with the possibility of group insurance. Finally, like in the literature on incomplete asset markets it considers imperfect insurability, but as in Radner (1982) it avoids the problem of unbounded asset demands by imposing direct restrictions. For proving existence of equilibrium we build on the Negishi format of welfare optimization (Negishi, 1960) extending it to account for the switches in active budget constraints caused by the bounds on insurability.

Overview

The paper proceeds as follows. Section 2 contains the model specification. It introduces the prevalence of illness and its dependence on goods that prevent illness. This generates the endogenous probabilities for individuals to incur specified diseases. Next, we turn to the specification of the behavioral model for the consumer-patient, who faces uncertainty with respect to incurring a disease as well as to access to treatment, and whose capacity to work and to enjoy leisure depends on the treatment received. We also include the capital market imperfection that some treatments may not be fully insurable. On the supply side, we treat services rendered by the health sector as regular production activities. Supply and demand are linked via commodity markets, while capacity constraints affect the probability of receiving timely service and effective treatment. Having completed the model specification we prove existence of general equilibrium and analyze model properties (section 3). Section 4 considers possible adaptations and extensions to allow for different types of uncertainty and for dynamic effects Section 5 discusses how health policies can be used to implement this solution and also considers more active interventions through subsidies, prohibition and equal access restrictions and section 6 concludes.

2. Model specification

Prevalence of illness

We distinguish I perfectly homogeneous consumer groups denoted by the index i , with a fixed number of members N_i . Every group i faces S uncertain illnesses (states), indexed s . In every state s , each consumer can undergo H treatments, indexed h . Uncertainty about treatment may be due to either limited access or limited effectiveness. We use these indices in fixed order ish , and as all summations also follow that order, we can suppose that the correspondences are restricted, i.e. that the subsets S_i and H_s of $\{1, \dots, S\}$ and $\{1, \dots, H\}$, define the illnesses that can be incurred by i , and the treatments applicable to illness s , respectively. Hence, $\sum_i \sum_s \sum_h x_{ish}$ or $\sum_{ish} x_{ish}$ can be read as shorthand for: $\sum_i \sum_{s \in S_i} \sum_{h \in H_s} x_{ish}$; similarly, $\sum_h x_{ish}$ is shorthand for: $\sum_{h \in H_s} x_{ish}$, all i and s ; but $\sum_h \mathbf{f}_h$ sums over all h . Furthermore, state $s=1$ with treatment $h=1$ refers to perfect health and no treatment and we assume that perfect health is defined for all i :

$$1 \in S_i \text{ for all } i$$

and non treatment is always possible:

$$1 \in H_s \text{ for all } s.$$

Prevalence of illness is expressed as the probability P_{is} of a member of group i , incurring illness s , while P_{ish} denotes the probability of incurring illness s and receiving treatment h . Clearly,

$$P_{is} = \sum_h P_{ish} \text{ and } \sum_s P_{is} = 1. \quad (2.1)$$

The illness probabilities are constrained by bounds:

$$\underline{P}_{is} \leq P_{is} \leq \bar{P}_{is}, \quad (2.2)$$

where the upper bound \bar{P}_{is} measures the fixed maximum potential for the illness. Even though illness may be viewed as a natural phenomenon, largely beyond the individual's control, we treat these probabilities as choice variables. This allows to distinguish various healthy states beyond $s=1$, say, with different occupations or place of residence but, more importantly, it allows the individual to opt for illness because of the financial or social benefits associated with this condition, and this is an essential facet of the insurability issue.

Prevention is seen as a production process including vaccination, education and provision of safe infrastructure (piped water, safe roads). It enters the model via adjustment of the lower bound on probabilities:

$$\underline{P}_{is}(d) = \hat{P}_{is} - q_{is}(d), \quad (2.3)$$

where the vector $d \in \mathfrak{R}_+^r$ denotes the use of inputs (commodities) for prevention (note that this vector is of fixed dimension, independent of ish). These inputs may include sanitation facilities, fresh air, safe roads and could, for instance, reflect a lack of clean water causing infectious diseases. Prevention is taken to shift the lower bound downward.

Assumption P (Prevention): The prevention functions $q_{is} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_+$, $q_{is}(d)$ are homogeneous, continuous, concave and non-decreasing.

The inputs in prevention are taken to be non-rival, i.e. to benefit all consumers simultaneously rather than being treated as consumer-group specific. Since addressing one disease could affect others we allow for cross effects among preventive measures. Note that \hat{P}_{is} can be taken to be zero for the healthy state if health is seen as the state of non-illness, that does not need prevention. Assumption P views prevention as a production process. Some of these entries in the input vector d could refer to fixed or exhaustible factors, such as fresh air, so as to represent the effect of environmental degradation on health. We distinguish between \hat{P}_{is} and \underline{P}_{is} mainly to represent the healthy state, where prevention is ineffective ($q_{il}(d) = 0$)

Consumer-patients

We now formulate a static decision model for the individual consumers or households as a utility maximization subject to a budget constraint. Utility $u_{ish}(x_{ish})$ depends on the commodity bundle $x_{ish} \in \mathfrak{R}_+^r$ consumed by consumer i in state s under treatment h , including pharmaceuticals, treatments and all other health related goods and services (inclusive of the patient's own time). The utility function satisfies:

Assumption C1 (utility): (a) Utility functions $u_{ish} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_+$, $u_{ish}(x_{ish})$ are strictly concave, homogeneous (with $u_{ish}(0) = 0$); (b) $\lim_{n \downarrow 0} n u_{ish}(x/n) = 0$ for any given x , and $u_{ish}(0/0) = 0$ by convention.

Functions such as Cobb Douglas and CES and their monotonic transformations satisfy this limit property. In addition, we must allow for satiation (and decreasing utility) in some states, precisely because uncured illness may lead to a situation where additional consumption fails to raise well being, and hence where additional transfers cannot compensate for lack of treatment. Typically, consumers will prefer health to illness, but

we do not yet make that assumption at this stage. The price vector does not carry the index s , indicating that, like in a lottery, we only consider idiosyncratic risk: all states s become effective simultaneously, and only the individuals face uncertainty as to which state will be theirs. In section 5 we indicate how the more general situation with aggregate risk can be dealt with.

Income in state sh is derived from a supply determined via a household production function $\mathbf{w}_{ish}(x_{ish})$ that depends on consumption, as in the efficiency wage literature, and from exogenously given² endowments \mathbf{g}_{ish} . This production function permits to depict the dependence of labor supply on consumption of food, education (e.g. on hygiene) and health related goods. It includes the production of effective time available for leisure or work.

Assumption C2 (household supply): The household production function satisfies $\mathbf{w}_{ish} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}^r$, $\mathbf{w}_{ish}(x_{ish}) = \mathbf{w}'_{ish} + \mathbf{w}^l_{ish}(x_{ish})$, where $\mathbf{w}^l_{ish}(\cdot)$ is continuous, concave, non-decreasing, homogeneous of degree one, and $\lim_{n \downarrow 0} n\mathbf{w}^l_{ish}(x_{ish}/n) = 0$ for any given x_{ish} , while $\mathbf{w}^l_{ish}(0/0) = 0$ by convention; the vector \mathbf{w}^0_{ish} is a constant.

This constant vector \mathbf{w}^0_{ish} may have negative entries, so as to allow for setup costs in labor production. It could also be taken as part of \mathbf{g}_{ish} . Restrictions on total endowments $\mathbf{w}_{ish}(x_{ish}) + \mathbf{g}_{ish}$ will be given in assumption C4 below.

At this point we introduce an indirect utility representation for compactness of notation. For fixed prices p , the utility of state sh satisfies:³

$$\begin{aligned} \tilde{u}_{ish}(p, v_{ish}) &= \max_{x_{ish} \geq 0} u_{ish}(x_{ish}) \\ & \text{s.t. } px_{ish} = p\omega_{ish}(x_{ish}) + p\gamma_{ish} + v_{ish}, \end{aligned} \quad (2.4)$$

where v_{ish} is the net insurance transfer for consumption in state sh and where we write $\tilde{u}_{ish}(p, v_{ish})$ instead of the more usual indirect utility function because net transfers rather than incomes are the relevant variables for insurance.

The individual is supposed to maximize expected utility subject to a budget constraint, with expected expenditure equal to expected income, taking all probabilities P_{ish} as given, and as true. Subjective probabilities could be incorporated via multiplication factors in the utility functions. Now the individual's problem with insurance can be written:

² In a dynamic interpretation of the model, the vector \mathbf{g}_{ish} includes initial stocks of capital goods.

³ The symbols after the \max refer to choice variables; px_{ish} denotes the inner product. Here the utility function is state-dependent but it could also be written without subscripts, as $u_i(x, s, h)$, to reflect that there is one system of preferences underlying choices in all states. Monotonic transformations can be used to control risk aversion.

$$\begin{aligned}
& \max_{v_{ish}, all\ sh} \sum_s P_{is} \sum_h \mathbf{p}_{ish} \tilde{u}_{ish}(p, v_{ish}) \\
& s.t. \quad \sum_s P_{is} \sum_h \mathbf{p}_{ish} (v_{ish} + t_i + \mathbf{f}_h) = 0 \\
& \quad \underline{a}_{ish} \leq v_{ish} + \mathbf{f}_h \leq \bar{a}_{ish},
\end{aligned} \tag{2.5}$$

where \mathbf{p}_{ish} denotes the conditional probability of treatment h given state s , and where the budget constraint indicates that the expected value of net transfers for consumption (v_{ish}) plus prevention charges (t_i) that are constant across s and h , and access to treatment (\mathbf{f}_h) is zero. The bounds on $(v_{ish} + \mathbf{f}_h)$, restrict the net flow of funds to and from sh and are therefore limits on insurability. We do not include the prevention charge in the bounds on transfer to reflect that it is possible to deny treatment but not to force a destitute and sick person to pay for prevention. In other words prevention costs are taken to be perfectly insurable while treatment costs are not. The bounds only relate to imperfections of the insurance system. The physical effects of inadequate or ineffective treatments can be dealt with through satiation of the utility function, and reduced productivity of consumption in the household supply function. While we view all insurance from the patient's perspective, the payments include compensation for the cost of treatment as well as disability, and may in part be financed from claims awarded in case of negligence of the medical staff. Hence, the bounds also reflect possible legal limits on the staff's liability. In this model the consumer eventually finances all claims. Doctors have perfect insurance and are only affected via the impact on the average price paid for their services. A natural extension would identify doctors and the risks they face as separate agents and might also highlight the dependence of the insurability on the patient's possession of human and physical capital. Here we merely postulate the bounds as functions of prices:

Assumption C3 (Insurability) For all $(s, h) \neq (I, I)$ bounds on insurability are defined by the functions $\underline{a}_{ish} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_-, \underline{a}_{ish}(p)$ and $\bar{a}_{ish} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_+, \bar{a}_{ish}(p)$. These functions are continuous and homogeneous of degree one. For $(s, h) = (I, I)$, there are no bounds: $\underline{a}_{iII}(p) = -\infty$ and $\bar{a}_{iII}(p) = \infty$.

The homogeneity assumption is made to avoid money illusion. The absence of bounds for healthy consumers with $h=I$ ensures that they can spend as much as they like on health insurance. Thus, in the model, limited insurability is expressed in three ways. First, consumers may choose a way of life when healthy that does not permit to participate in insurance schemes, as they might, say, have no fixed place of residence (i.e. for $s=I, h \neq I$). Second, insurability can be limited for specific medical treatments (for $s \neq I, h \neq I$). Finally, even the disability compensation may be restricted (for $s \neq I, h=I$).

Endogenous probabilities: prevention and access to treatment

Whereas the role played in the health sector by patients, drugs and doctors seems obvious, the function of health insurance organizations may need clarification. When local communities no longer have the means to provide and finance all necessary treatments, the health insurance organization becomes a key intermediary between consumers and producers for access to health goods. It collects the premiums and pays the bills of doctors and hospitals. It also keeps track of the probabilities of occurrence of illness and the availability of treatment capacity, to inform consumers on these probabilities and to manage the lotteries that decide about who is treated within the time period under consideration. Based on these probabilities, healthy individuals decide, given their own budget, time preference and risk aversion, about the treatments they want to be covered, the sickness benefits they want to receive, and associated to this the amount they want to spend on premiums. Like with contributions to a fire brigade, the willingness to maintain treatment capacity determines the scale and quality of the services available in case of emergency and also the probability to find no service available when the need arises.

The bounds on insurance constrain the choice but there is perfect within group insurability in the sense that no member is excluded. This may not seem very restrictive because all group members are taken to be identical and it is always possible to account for lack of insurability between specified individuals by distinguishing a large number of groups, as there is no insurability across groups. Yet, we treat the number of individuals as divisible, and this is only warranted if they are not too small. Moreover, we suppose below that for each group all uncertain events materialize within the (single) period under consideration and this also is only acceptable if groups are large. In section 4 we return to this issue, allowing for further uncertainty, over time and with respect to verification of sickness s and adequacy of treatment h .

We assume that the insurance organization acts in the interest of the consumers and maximizes their expected utility at group level. It might be a mutual or public insurance organization, or a private company facing sufficient competition. Since the insurer acts on his behalf, the consumer is willing to accept the choices on treatment, given the prevailing bounds on insurability. Because of this, the assignment of patients to treatments can be considered the result of a lottery whereby the insurer only registers the individual applications.⁴ For the insurer, the probabilities of access to treatment become endogenous. The lottery has probabilities $p_{ish} = n_{ish} / N_{is}$, where n_{ish} is the number of individuals of group i with illness s receiving treatment h , and $N_{is} = \sum_h n_{ish} = P_{is} N_i$ is the number of individuals from group i with illness s . This measurement of probability by itself implies that all states are realized simultaneously. Also, the prevalence P_{is} is treated as an endogenous choice variable to reflect the notion that every individual will choose

⁴ Here the cost of diagnosis is taken to be part of consumption in state s and not part of the treatment. Quality differences in diagnosis and treatment can be considered part of the state definition.

the preferred health status (possibly illness if the benefits are attractive). The conditional probability \mathbf{p}_{ish} will be such that the expected utility of every member is maximized. In other words, the insurer allocates group members to state sh so as maximize individual expected utility (which is identical for all group members) subject to constraints on prevalence \underline{P}_{is} and P_{is} from (2.1), while meeting the budget constraint, inclusive of the given charges for prevention and access to treatment:

$$\begin{aligned}
& \max_{N_{is}, P_{is}, n_{ish}, \pi_{ish} \geq 0, v_{ish}, \text{all } sh} \sum_s P_{is} \sum_h \pi_{ish} \tilde{u}_{ish}(p, v_{ish}) \\
& \text{s.t.} \quad \sum_s P_{is} \sum_h \pi_{ish} (v_{ish} + t_i + \phi_h) = 0 \\
& \quad \underline{a}_{ish} \leq v_{ish} + \phi_h \leq \bar{a}_{ish} \\
& \quad \underline{P}_{is} \leq P_{is} \leq \bar{P}_{is} \\
& \quad P_{is} = \frac{N_{is}}{N_i} \\
& \quad \pi_{ish} = \frac{n_{ish}}{N_{is}} \\
& \quad N_{is} = \sum_h n_{ish} \\
& \quad \sum_s N_{is} = N_i.
\end{aligned} \tag{2.6}$$

In this program the cost f_h of access to treatment, associated to prevailing capacity constraints, affects the assignment of patients and hence, the probabilities \mathbf{p}_{ish} . If capacity is limited, the probability of receiving treatment will be low. Thus, even though the charge f_h can be interpreted as the outcome of a rationing process, the assignment is not necessarily an inefficient since the capacity itself reflects consumer preferences.

It would be natural to expect the upper bound P_{is} to be non-binding for all $s > 1$, since health is typically preferred to illness, but we did not impose sufficient restrictions to this effect. If every consumer chooses to be healthy, all P_{is} will be at lower bound for $s > 1$. We also note that for fixed probabilities P_{is} and \mathbf{p}_{ish} program (2.6) reduces to (2.5).

Because of the multiplication by endogenous probabilities program (2.6) is not convex in its choice variables. It is shown in the next section that it has a single stationary point nonetheless, that defines its global optimum and that its response to parametric changes possesses the appropriate continuity properties. This enables us to work with endogenous probabilities like with a regular demand.

Finally, we require a specification for sharing the cost of access prevention. Solidarity transfers across groups are readily incorporated, but here we assume that every group i pays for itself, i.e. is being charged

$$t_i = \mathbf{y}_i d \tag{2.7}$$

where d is the input in prevention defined earlier. Thus, the financial contribution to prevention is independent of s . To measure the willingness-to-pay for prevention,⁵ we include explicit group specific prevention measures d_i , even though their assumed non-rivalry across groups implies that $d_i = d$.

$$\begin{aligned}
& \max_{d_i \geq 0, N_{is}, P_{is}, n_{ish}, \pi_{ish} \geq 0, t_i, v_{ish}, \text{all } sh} \sum_s P_{is} \sum_h \pi_{ish} \tilde{u}_{ish}(p, v_{ish}) \\
& \text{s.t.} \quad \sum_s P_{is} \sum_h \pi_{ish} (v_{ish} + t_i + \phi_h) = 0 \quad (\lambda_i) \\
& \quad t_i = \psi_i d_i \\
& \quad \underline{a}_{ish}(p) \leq v_{ish} + \phi_h \leq \bar{a}_{ish}(p) \\
& \quad \underline{P}_{is}(d_i) \leq P_{is} \leq \bar{P}_{is} \\
& \quad P_{is} = \frac{N_{is}}{N_i} \quad (2.8a) \\
& \quad \pi_{ish} = \frac{n_{ish}}{N_{is}} \\
& \quad N_{is} = \sum_h n_{ish} \\
& \quad \sum_s N_{is} = N_i \\
& \quad d_i = d \quad (\tilde{\psi}_i)
\end{aligned}$$

enabling us to compute the willingness-to-pay for prevention from

$$y_i = \frac{\tilde{y}_i}{I_i}, \quad (2.8b)$$

where the division by the (positive) marginal utility of income I_i performs the conversion to monetary units. Clearly, this Lindahl pricing is a difficult procedure, and government intervention may be required to co-ordinate the decisions of the different groups.

Production

The production side distinguishes three kinds of production activities: (1) firms producing drugs or other commodities; (2) firms supplying health facilities (e.g., physicians and hospitals); (3) firms controlling prevention (e.g., water treatment plants).

First, commodity producing firms follow a standard specification. Every firm j maximizes profits subject to a technology constraint represented by a transformation function $F_j(y_j)$.

$$\max_{y_j} \{ py_j \mid F_j(y_j) \leq 0 \}. \quad (2.9)$$

⁵ Symbols in brackets on the right hand side denote Lagrange multipliers.

The transformation function satisfies:

Assumption S1 (Production of commodities): The transformation function $F_j : \mathfrak{R}' \rightarrow \mathfrak{R}, F_j(y_j)$ is strictly quasiconvex, nonstationary, non-decreasing, and homogeneous of degree one.

The homogeneity assumption implies that optimal profits are zero in equilibrium, that net supply is only determined up to a scalar, and that zero is a feasible net output. Hence, consumers who own firms receive their reward via the price of their endowments (that include all capital goods), and there is no need to channel profits to them. The requirement that the function should be nonstationary and nondecreasing implies that feasible vectors cannot be strictly positive. We also need a standard assumption to ensure boundedness of supply in the economy.

Assumption S2 (Boundedness of output): Every feasible vector y_j has a negative entry for least one input commodity that is not being produced, or via its inputs directly or indirectly requires a commodity that is not being produced.

Second, service providers – typically physicians and hospitals – determine the treatment capacity according to the production function $N_h(c)$ using inputs c , so as to maximize their profits, for a given rental price f_h on access to treatment capacity and price of inputs p :

$$\max_{c \geq 0} \sum_h f_h \bar{N}_h(c) - pc. \quad (2.10)$$

This rental price is determined as a clearing price on a competitive market.

Assumption S3 (Treatment capacity): The treatment capacity function $\bar{N}_h : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_+, \bar{N}_h(c)$ is continuous, concave, nondecreasing, and homogeneous of degree one.

This specification is somewhat restrictive. All treatments are taken to depend on common inputs c to reflect the cross effects in hospitals and other health facilities. Assumption S3 ensures that the profit will be nonnegative, and neglects setup costs.⁶ Homogeneity of degree one is maintained to circumvent explicit profit distribution.

Third, to describe prevention we rephrase assumption P. We define aggregate prevention as $Q_{is}(d) = q_{is}(d)N_i$, for $q_{is}(d)$ defined as in assumption P. This permits to formulate prevention through a regular production function:

⁶This admittedly rudimentary specification is chosen for ease of exposition. Setup costs could be introduced, as a committed expenditure. However, the decision whether to incur these costs would amount to a discrete choice and hence introduce a non-convexity.

Assumption S4 (Prevention): The prevention functions $Q_{is} : \mathfrak{R}_+^r \rightarrow \mathfrak{R}_+, Q_{is}(d)$ are continuous, concave, non-decreasing, and homogeneous of degree one.

This is as in assumption P. Homogeneity implies that prevention may exceed the requirements but clearly, probabilities will remain nonnegative.

Market equilibrium and feasibility

Commodity balances link supply with demand. Here we maintain the assumption that all states materialize simultaneously within the time period under consideration. This is reflected in a fully consolidated commodity balance, with an associated shadow price p independent of s and h .

$$\sum_{ish} n_{ish} x_{ish} + c + d = \sum_{ish} n_{ish} (\mathbf{w}_{ish}(x_{ish}) + \mathbf{g}_{ish}) + \sum_j y_j, \quad (2.11)$$

and p is the associated, state independent market price. Furthermore, we must also impose the constraints that clear the markets for treatment.

$$\sum_{is} n_{ish} \leq \bar{N}_h(c), \quad (2.12)$$

with $\mathbf{f}_h \geq 0$ as associated price for rental of treatment capacity that is taken to be zero if the bound is not binding and where equality holds whenever the price is positive. Furthermore, a price normalization has to be imposed:

$$\sum_k p_k = \mathbf{k}, \quad (2.13)$$

for given positive value of \mathbf{k} (to be specified below following the normalization of welfare weights).

Budgets

Having completed the model specification we briefly discuss the resulting budget constraints. The government budget redistributes the cost of prevention and access to treatment:

$$pc + pd = \sum_{ish} n_{ish} (t_i + \mathbf{f}_h)$$

while total net transfers sum to zero:

$$\sum_{ish} n_{ish} (v_{ish} + t_i + \mathbf{f}_h) = 0,$$

that can be rewritten as:

$$\sum_i N_i \sum_s P_{is} \sum_h \mathbf{p}_{ish} (v_{ish} + t_i + \mathbf{f}_h) = 0,$$

implying that total expected individual transfers sum to zero as well.

Feasibility

Finally, to ensure existence of equilibrium, we need specific feasibility assumptions:

Assumption C4 (Feasibility): For $i=1$ referring to group 1, $s=1$ to the healthy condition and $h=1$ to non treatment: (a) $P_{i1} > 1$; (b) $\sum_s \dot{P}_{is} < 1$; (c) $\bar{N}_1(0) = \infty$; $\mathbf{f}_1 = 0$; (d) $\mathbf{w}_{is1}(0) + \mathbf{g}_{is1} \geq 0$; (e) $\mathbf{w}_{ilh1}(0) + \mathbf{g}_{ilh1} \geq 0$, with strict inequality for some commodity k ; (f) $\mathbf{w}_{111}(0) + \mathbf{g}_{111} > 0$; (g) $u_{111}(x_{111})$ is locally non-satiated; (h) $u_{111}(x_{111})$ is increasing.

Here, condition (a) implies that constraints on health are only due to specified illnesses and absence of cures; (b) that there is room for choice between healthy and unhealthy states; (c) that there is no bound on the capacity of no treatment, and obviously the price charged is zero; (d) that autarky without consumption and with zero transfers is feasible in all states, due to the possibility of non treatment; (e) that every consumer group can, if healthy, supply some commodity from labor or from stocks, with zero consumption; (f) that consumer group 1 (which may be of arbitrarily small size) can supply positive amounts of everything; finally (g) that healthy consumers can always spend excess income as their point of satiation, if any, is not attainable for the given supply potential of the economy;. We note that the autarky-related assumptions (d)-(f) only ensure feasibility in a formal sense, as they admit illnesses being left without treatment, as a possible reflection of institutional incapacity.

3. Equilibrium: existence, Pareto efficiency and other properties

We are now ready to define the equilibrium of this model, which we refer to as an Arrow Debreu equilibrium since it characterizes prices at which the total supply and demand of the respective agents satisfy commodity balances.

Definition 3.1 (Arrow Debreu equilibrium): An Arrow Debreu equilibrium of the health model is a nonnegative vector of prices $(p, \mathbf{f}, \mathbf{y})$ normalized according to (2.13), such that, with producer behavior (2.9)-(2.10) and consumer-insurer behavior (2.8), the balances (2.11), (2.12) are satisfied.

The proof of existence and efficiency will make use of the well known conversion to the Negishi format of welfare optimization (Negishi, 1960). There are, however, two major complications: the non-convexity of the consumer problem, and the bounds on insurability.

For given positive welfare weights \mathbf{a}_{ish} , we construct the social welfare function as the weighted sum of utilities $n_{ish}u_{ish}$ expressed as probabilities:

$$d_i = d \quad (\Psi_i)$$

The objective in (3.1) deviates from a weighted sum of individual expected utilities of individuals in group i since it has state specific ‘welfare’ weights \mathbf{a}_{ish} rather than individual welfare weights. These reflect the effect of the bounds on insurability that do not appear in (3.1). The remaining constraints repeat earlier transformation functions and definitions of probabilities.

Associated to the welfare weights of this program are budget constraints. These are not part of the program itself but the equilibrium finds values for the welfare weights that ensure that they are being satisfied at prevailing shadow prices. The institutional arrangements for financing the health system finds expression in these budgets.

We must account for the constraints on transfers implied by the bounds on insurability. Because of these, we cannot calculate the budget deficit for every group i , because this depends on which bounds are binding. Hence, rather than directly calculating the usual budget deficit like of the Negishi approach we formulate a welfare program determining the transfers at given prices and welfare weights, and from this we derive the budget deficits. The program reads:⁷

$$\begin{aligned} & \max_{n_{ish} \geq 0, v_{ish}, all\ ish} \sum_i \delta_i \sum_{sh} n_{ish} \tilde{u}_{ish}(p, v_{ish}) \\ & s.t. \quad \sum_{ish} n_{ish} (v_{ish} + \Psi_i d + \phi_h) = 0 \\ & \quad n_{ish} (v_{ish} + \phi_h) \leq n_{ish} \bar{a}_{ish}(p) \quad (\mu_{ish}) \\ & \quad n_{ish} \tilde{u}_{ish}(p, v_{ish}) \geq n_{ish} \underline{u}_{ish} \quad (\sigma_{ish}) \\ & \quad \underline{P}_{is}(d) \leq \frac{\sum_h n_{ish}}{N_i} \leq \bar{P}_{is} \\ & \quad \sum_{sh} n_{ish} = N_i, \end{aligned} \quad (3.2)$$

for given

$$\mathbf{d}_i \geq 0, p, \mathbf{f}_h, w_{ish}, d$$

and

$$\underline{u}_{ish} = \tilde{u}_{ish}(p, \max(\underline{a}_{ish}(p) - \mathbf{j}_h, -p\mathbf{w}_{ish}(x_{ish}) - p\mathbf{g}_{ish})).$$

⁷ Alternatively, building on the ‘full’ format (GK, chapters 3 and 12) we could insert the budget and insurability constraints within the welfare program itself and iterate between Lagrange multipliers and the given prices in these budgets.

“New” welfare weights can be determined on the basis of the budget surplus from this program and the prevailing shadow prices $\mathbf{m}_{ish}, \mathbf{s}_{ish}$:

$$b_i = \max(-\sum_s \sum_h n_{ish} (v_{ish} + \mathbf{y}_i d + \mathbf{f}_h), 0) \quad (3.3)$$

$$\hat{\mathbf{d}}_i = \mathbf{r}(\mathbf{d}_i + b_i) \quad (3.4)$$

$$\sum_i \hat{\mathbf{d}}_i = 1 \quad (3.5)$$

$$\hat{\mathbf{a}}_{ish} = (\hat{\mathbf{d}}_i + \mathbf{s}_{ish}) / (1 + \mathbf{m}_{ish}) \quad (3.6)$$

where \mathbf{r} is a positive scaling factor for normalization of welfare weights, which only includes the healthy state. Note that adjustment (3.6) is continuous. Treating lower bounds on insurability in (3.2) via the utility function permits to keep $\hat{\mathbf{a}}_{ish}$ nonnegative and to make it positive whenever $\hat{\mathbf{d}}_i > 0$.

Definition 3.2 (Negishi equilibrium): A Negishi equilibrium of the health model is a nonnegative vector of welfare weights normalized according to (3.5), such that, with welfare program (3.1) and transfer allocation (3.2) and welfare weight update (3.3), (3.4), (3.6), budget deficits are zero for all i and the equalities $\mathbf{d}_i = \hat{\mathbf{d}}_i$ and $\mathbf{a}_{ish} = \hat{\mathbf{a}}_{ish}$ hold.

Note that if in equilibrium some bound on transfers is effective, and thus some multiplier μ_{ish} or σ_{ish} not equal to zero, the sum of welfare weights α_{ish} is not equal to one, while if all insurability bounds are ineffective in equilibrium the welfare weights are equal to δ_i , independent of state s and h and sum up to one. In the latter case, the objective of (3.1) becomes a social welfare function of expected utility.

Proposition 1 (Existence): Let assumptions C1-C4 hold for every consumer group i , assumptions S1-S2 for every firm j , and assumptions S3-S4 for health care and prevention. Then,

1. there exists a Negishi equilibrium as Definition 3.2;
2. in equilibrium all commodity prices are positive and every consumer group i has positive income.
3. this is an Arrow Debreu equilibrium as in Definition 3.1;

Proof.

Assertion (1). The proof proceeds as follows. (i) We start by converting programs (3.1) and (3.3) into a convex program, by appropriate redefinition of variables; next, we closely mimic the proof in Negishi format of Ginsburgh and Keyzer, 1997 (GK for short) chapter 3; we show that (ii) the programs are feasible and satisfy constraint qualification; (iii) they are bounded; (iv) commodity prices obtained as Lagrange multipliers from the welfare program are not all zero; (v) the mapping for generating the new welfare weights

has a fixed point; (vi) in the fixed point budget deficits are zero; (vii) the stated equalities on welfare weights belong to the equilibrium.

(i) *Convexity.* We rewrite the welfare program in aggregate terms, writing all probabilities in terms of n_{ish} and defining aggregate consumption as $X_{ish} = n_{ish} x_{ish}$:

$$\begin{aligned}
& \max_{c,d,d_i,X_{ish} \geq 0, n_{ish} \geq 0, y_j, \text{ all } ish} \sum_{ish} n_{ish} \alpha_{ish} u_{ish}(X_{ish}/n_{ish}) & [P1] \\
& \text{s.t.} \quad \sum_i X_{ish} + c + d \leq \sum_{ish} n_{ish} (\omega_{ish}(X_{ish}/n_{ish}) + \gamma_{ish}) + \sum_j y_j & (p) \\
& \quad F_j(y_j) \leq 0 \\
& \quad \underline{P}_{is}(d_i) N_i \leq \sum_h n_{ish} & (\eta_{is}) \\
& \quad \sum_h n_{ish} \leq \bar{P}_{is} N_i & (\chi_{is}) \\
& \quad \sum_{is} n_{ish} \leq \bar{N}_h(c) & (\phi_h) \\
& \quad \sum_{sh} n_{ish} \leq N_i & (\zeta_i) \\
& \quad d_i = d & (\psi_i)
\end{aligned}$$

The concavity of the “extended” functions $U_{ish}(X_{ish}, n_{ish}) = n_{ish} u_{ish}(X_{ish}/n_{ish})$ for utility and $\Omega_{ish}(X_{ish}, n_{ish}) = n_{ish} \omega_{ish}(X_{ish}/n_{ish})$ for endowments follows from the homogeneity and limit requirements in assumptions C1 and C2 (see Theorem 1.5 in Appendix A of GK). Convexity of other constraints on production, prevention, and treatment follows from assumptions S1-S2, S3, S4 respectively. Next, we transform program (3.2) after defining aggregate transfers $V_{ish} = n_{ish} v_{ish}$ and writing aggregate indirect utility as

$$\begin{aligned}
\tilde{U}_{ish}(p, V_{ish}, n_{ish}) &= \max_{X_{ish} \geq 0} U_{ish}(X_{ish}, n_{ish}) \\
& \text{s.t.} \quad pX_{ish} = p\Omega_{ish}(X_{ish}, n_{ish}) + p\gamma_{ish} n_{ish} + V_{ish}. & [P2]
\end{aligned}$$

This function is jointly concave and homogeneous of degree one (V_{ish}, n_{ish}) . We substitute it in program (3.3) and this yields:

$$\begin{aligned}
& \max_{n_{ish} \geq 0, V_{ish}, \text{ all } ish} \sum_i \mathbf{d}_i \sum_{sh} \tilde{U}_{ish}(p, V_{ish}, n_{ish}) & [P3] \\
& \text{s.t.} \quad \sum_{ish} (n_{ish} (t_i + \mathbf{f}_h) + V_{ish}) = 0 & (\mathbf{l}) \\
& \quad n_{ish} \mathbf{f}_h + V_{ish} \leq n_{ish} \bar{a}_{ish}(p) & (\mathbf{m}_{ish}) \\
& \quad \tilde{U}_{ish}(p, V_{ish}, n_{ish}) \geq n_{ish} \underline{u}_{ish} & (\mathbf{s}_{ish}) \\
& \quad \underline{P}_{is}(d) \leq \frac{\sum_h n_{ish}}{N_i} \leq \bar{P}_{is} \\
& \quad \sum_{sh} n_{ish} \leq N_i,
\end{aligned}$$

for given p, \mathbf{f}_h, t_i, d and $\underline{u}_{ish} = \tilde{u}_{ish}(p, \max(\underline{a}_{ish}(p) - \mathbf{j}_h, -p\mathbf{w}_{ish}(x_{ish}) - p\mathbf{g}_{ish}))$. This program is convex since it has a concave objective and convex constraints.

(ii) *Feasibility and constraint qualification.* To check feasibility of [P1], we assign $y_j = 0, c = 0, d = 0$, and award zero consumption to all consumers except consumer group 1, while moving the maximum number of consumers to healthy state, and to no treatment. By assumption C4(a)-(d), and homogeneity of the prevention function S4, this allocation is feasible in [P1]. Slater's constraint qualification holds because by C4(b) the healthy state ($s=1$) is attainable and by C4(f) it leaves room for positive consumption and relaxation of all bounds on prevention treatment; by C4(a)-(b) there is room for moving n_{ish} away from all its bounds. Hence, all Lagrange multipliers are well defined. For program [P3], we can also ensure feasibility, because regarding n_{ish} , the healthy state can, by C4(a)-(b), always accommodate the surplus population, and regarding transfers, by C4(d), abstaining from treatment $V_{is1} + n_{is1}t_i = 0$ is feasible ($\mathbf{f}_1 = 0$). This possibility of making all inequality constraints nonbinding ensures boundedness of the multipliers.

(iii) *Boundedness of allocations.* That n_{ish} is bounded is obvious in both [P1] and [P3]. In program [P1], supply and demand are bounded, by assumption S2 on production; and in [P3], by assumption C3 on the bounds, and by the budget constraint, V_{ish} is bounded as well.

(iv) *Nonzero commodity prices.* Welfare weight normalization (3.5) implies that the weight \mathbf{d}_i will be positive for some i . This leads by (3.6) to a positive welfare weight for all sh of that group, including $s=1$ and $h=1$, which by C4(a)-(b) has the possibility of receiving a positive number of individuals. Then in [P1], by nonsatiation C4(g), if all other treatments for $s=1$ are satiated, consumers will move to $h=1$. Hence, some commodity price p_k must be positive.

(v) *Fixed point mapping.* Denoting the associated variables by $\{\cdot\}$, we remark that program [P1] defines a correspondence $\{\mathbf{a}\} \Rightarrow \{X, n, d\} \times \{p, \mathbf{f}, \mathbf{y}\}$, and if through [P2] X_{ish}, n_{ish} and p we replace V_{ish} , this program defines a correspondence $\{\mathbf{d}\} \times \{X, n, d\} \times \{p, \mathbf{f}, \mathbf{y}\} \Rightarrow \{\mathbf{m}, \mathbf{s}\}$. Finally, also after substituting out transfers, (3.3)-(3.6) define a continuous mapping $\{\mathbf{d}\} \times \{X, n, d\} \times \{p, \mathbf{f}, \mathbf{y}\} \times \{\mathbf{m}, \mathbf{s}\} \rightarrow \{\mathbf{d}\} \times \{\mathbf{a}\}$. Substituting this into the correspondence for [P1] to replace \mathbf{a} yields, jointly with [P3]: $\{\mathbf{d}\} \times \{X, n, d\} \times \{p, \mathbf{f}, \mathbf{y}\} \times \{\mathbf{m}, \mathbf{s}\} \Rightarrow \{\mathbf{d}\} \times \{X, n, d\} \times \{p, \mathbf{f}, \mathbf{y}\} \times \{\mathbf{m}, \mathbf{s}\}$. Regarding ranges and domains, we constrain the weights \mathbf{d} to the simplex \mathbf{D} satisfying (3.5), and define closed intervals $[\overline{X}, n], [\overline{p}, \mathbf{f}, \mathbf{y}], [\overline{\mathbf{m}}, \mathbf{s}]$ for the respective variables, with zero lower bounds and upper bounds so high that no element of it can be reached. For $[\overline{X}, n]$, the existence of such an upper bound follows from S2, and for $[\overline{p}, \mathbf{f}, \mathbf{y}]$ and $[\overline{\mathbf{m}}, \mathbf{s}]$ from constraint qualification (see (ii)). Thus, the domain $\mathbf{D} \times [\overline{X}, n] \times [\overline{p}, \mathbf{f}, \mathbf{y}] \times [\overline{\mathbf{m}}, \mathbf{s}]$ is compact convex. From (i)-(iii) and the maximum theorem follows that this correspondence is uppersemicontinuous, compact convex-valued. It remains to verify that it maps into itself. For the variables in

$[\overline{X}, n], [\overline{p}, \overline{\mathbf{f}}, \overline{\mathbf{y}}], [\overline{\mathbf{m}}, \overline{\mathbf{s}}]$, this is implied by the construction of the interval; for those in \mathbf{D} , this follows from (3.4). Therefore, by Kakutani's Theorem, a fixed point exists.

(vi) *Zero budget deficits.* By constant returns in S3 and S4, the charges $t_i + \mathbf{f}_h$ paid by n_{ish} in total cover the full cost of treatment and prevention $pc + pd$. Hence, by the complementary slackness condition of the commodity balance in [P1], budget surpluses cannot be positive for all i . Then, by (3.3) they must be zero in the fixed point.

(vii) *Welfare weight as equalities.* From (3.4) and (vi) follows that $\mathbf{r} = \mathbf{1}$ and $\mathbf{d}_i = \hat{\mathbf{d}}_i$. Note that by price normalization $\mathbf{1} = \mathbf{1}$ holds in the fixed point, and that, in [P3], v_{ish} solves $\max_{v_{ish}} [(\hat{\mathbf{d}}_i + \mathbf{s}_{ish}) \tilde{u}_{ish}(p, v_{ish}) - (\mathbf{1} + \mathbf{m}_{ish}) v_{ish}]$, whenever $n_{ish} > 0$. Then, replacing \mathbf{d}_i by substitution of $\hat{\mathbf{a}}_{ish}$ in the objective of [P3] while dropping the bounds on transfers yields a solution that agrees with [P1] and (3.6). Hence, $\hat{\mathbf{a}}_{ish} = \mathbf{a}_{ish}$ belongs to the equilibrium.

Assertion 2. Since some commodity price is positive, consumer group $i=1$ has positive income and as some of its members are in healthy state, where by C4(h) and the argument of (iv) above, their utility function is increasing in all commodities, all prices p are positive and consumers have positive income. Therefore, all consumers must have positive welfare weight: if they had zero weight, their full consumption bundle would accrue to consumer 1, and since all prices are positive they would have a budget surplus. Finally, positive prices, by complementary slackness in [P1], also imply that commodity balances (3.1) hold with equality.

Assertion 3. Full correspondence to an Arrow Debreu equilibrium follows from the Lagrangean inequality applied to the welfare program, exactly as in the standard proof of the Second Welfare Theorem (Proposition 1.9 in GK), while (vi) implies that for every i , budget deficits are zero.

At this point a remark on computation may be in order. The model of proposition 1 has many dimensions, witness the wide array of subscripts (even though the restrictions S_i and H_s may be used to limit the number of possible combinations). In the competitive model, if the number of commodities is small and the number of consumers large, the dimensionality problem can be addressed via stochastic optimization techniques that adjust prices based on individual sampling of excess demands among consumers (Ermoliev et al., 2000). Here it would seem that the extended Negishi format [P1]-[P3] offers the suitable computational framework and that sampling techniques could be applied to approximate the optimum of the welfare programs, in case these become too large to be solved by deterministic methods.

We pursue our characterization of the model with two propositions that focus on the choice of treatment, and hence on the mechanism generating the probabilities. For this, we define the potential consumer surplus (or conjugate utility):

$$\hat{u}_{ish}(p, \mathbf{I}) = \max_v \tilde{u}_{ish}(p, v) - \mathbf{I}v \quad (3.7)$$

This function can be used to characterize the attractiveness of states and treatments (division by marginal utility of expenditure \mathbf{I} expresses the consumer surplus in money metric). For states sh whose bounds on insurability are not effective, the optimal v in (5.1) will be an equilibrium value and also be optimal in (3.2).

Proposition 2 (Sickness) Consider the allocations from program (3.2), in equilibrium. If $\hat{u}_{ish}(p, \mathbf{I}_i) < \hat{u}_{i11}(p, \mathbf{I}_i)$ for $s \neq 1$, and $\mathbf{I}_i = \mathbf{I} / \mathbf{d}_i$, then for all $s \neq 1$: $P_{is} = \underline{P}_{is}(d)$ and $\mathbf{h}_{is} > 0$ whenever $P_{is} > 0$.

Proof. We consider the derivative of the Lagrangean of (3.2) w.r.t. n_{ish} (it exists even if none of the utility, and technological relationships in the model are differentiable). In equilibrium the multiplier associated to the aggregate budget will be equal to unity (because by (2.13) prices follow the normalization of welfare weights). Now, if $P_{is} = 0$ it is equal to the lower bound by construction. Otherwise, suppose that $\mathbf{h}_{is} = 0$ for some $s \neq 1$. Then, for some h , $\mathbf{d}_i \tilde{u}_{ish} - v_{ish} = \mathbf{f}_h + \mathbf{c}_{is} + t_i + \mathbf{z}_i$, whereas the healthy state without treatment has $\mathbf{d}_i \hat{u}_{i11} = \mathbf{d}_i \tilde{u}_{i11} - v_{i11} \leq -\mathbf{h}_{i1} + t_i + \mathbf{z}_i$. Under the condition assumed in the proposition, $\mathbf{d}_i \hat{u}_{i11}(p, \mathbf{I}_i) - \mathbf{d}_i \hat{u}_{ish}(p, \mathbf{I}_i) > 0$. Hence, $0 < \mathbf{d}_i \hat{u}_{i11}(p, \mathbf{I}_i) - \mathbf{d}_i \tilde{u}_{ish} + v_{ish} \leq -\mathbf{h}_{i1} - \mathbf{f}_h - \mathbf{c}_{is}$, a contradiction. Hence, $\mathbf{h}_{is} > 0$ and by complementary slackness $P_{is} = \underline{P}_{is}(d)$.

The proposition could be interpreted as defining true sickness, i.e. a condition that will never be opted for by own choice, because it is less desirable than health. In a similar vein, the next proposition defines inferior treatments that will never be selected because of their physical deficiencies or their limited insurability. It also highlights the associated pattern of specialization to treatments. To account for limits on insurability we define the actual consumer surplus, as opposed of the potential surplus (3.7):

$$\hat{\hat{u}}_{ish}(p, \mathbf{I}, \underline{v}, \bar{v}) = \max_{\underline{v} \leq v \leq \bar{v}} \tilde{u}_{ish}(p, v) - \mathbf{I}v. \quad (3.8)$$

This enables us to define the (linear) assignment program:

$$\begin{aligned}
& \max_{n_{ish} \geq 0, \text{all } ish} \sum_i \mathbf{d}_i \sum_{sh} n_{ish} \mathbf{u}_{ish} \\
& \text{s.t.} \quad \underline{P}_{is}(d) N_i \leq \sum_h n_{ish} \leq \bar{P}_{is} N_i \\
& \quad \sum_{sh} n_{ish} \leq N_i \\
& \quad \sum_i \sum_s n_{ish} \leq \bar{N}_h(c),
\end{aligned} \tag{3.9}$$

for given $\mathbf{u}_{ish} = \hat{u}_{ish}(p, l / \mathbf{d}_i, \underline{a}_{ish} - \mathbf{f}_h, \bar{a}_{ish} - \mathbf{f}_h)$. The reformulation highlights that money metric consumer surplus rather than utility itself is the relevant driver. It also illustrates that the assignment of patients can be viewed as an optimal migration problem that, because of its linearity, will specialize toward the preferred treatments until full capacity is reached. Conversely, inferior treatments will be eliminated.

Proposition 3 (Elimination of inferior treatments) Consider the allocations from program (3.10), in equilibrium. If for $h \neq h'$, $\hat{u}_{ish'}(p, \mathbf{l}_i) < \hat{u}_{ish}(p, \mathbf{l}_i)$, $\bar{a}_{ish'}(p) < \bar{a}_{ish}(p)$ and $\underline{a}_{ish'}(p) > \underline{a}_{ish}(p)$, then $n_{ish'} = 0$.

Proof. By the same line of argument as for proposition 3, income is always better spent in treatment h than in h' .

We note with reference to (3.8) that as individuals of group i become richer, their marginal utility \mathbf{l}_i will drop and \mathbf{u}_{ish} will generally rise. However, for treatments whose utility level approaches satiation, the increase in surplus will be less, and consequently, these treatments disappear from the set of selected activities. This will lead to an elimination of inferior treatments. Similarly, treatments with effective upper bounds on insurability will disappear with falling \mathbf{l}_i , since they have no scope for raising v_i .

4. Extending the model

This section discusses possible extensions of the model especially to allow for different kinds of uncertainty, prevalence and prevention.

Disaster

It has been argued that expected utility maximization may not be the relevant criterion in issues of life and death, since it does not attribute sufficient weight to extreme events. To account for this, the objective can be extended with a penalty term that attributes

additional weight to such events, i.e. to situations where utility drops below a critical level:⁸

$$\begin{aligned}
& \max_{v_{ish}, \text{all } sh} \sum_s P_{is} \sum_h \mathbf{P}_{ish} [\tilde{u}_{ish}(p, v_{ish}) - \mathbf{b} \max(\underline{u}_{ish} - \tilde{u}_{ish}(p, v_{ish}), 0)] \\
& \text{s.t.} \quad \sum_s P_{is} \sum_h \mathbf{P}_{ish} (v_{ish} + t_i + \mathbf{f}_h) = 0 \\
& \quad \underline{a}_{ish} \leq v_{ish} + \mathbf{f}_h \leq \bar{a}_{ish},
\end{aligned} \tag{4.1}$$

where \mathbf{b} is the penalty factor. However, since the term in square brackets can be viewed as a (concave) state dependent “indirect” utility function in its own right, the specification seems flexible enough to attribute a high weight to risky situations.

Uncertainty about the effectiveness of treatment

So far, the only uncertainty about the treatment is whether the lottery will be favorable, that is whether treatment h will be accessed. The model disregards all uncertainty about the effectiveness of the treatment itself. The probability of the access lottery depends fully on capacity and demand for treatment, and the patient choose the best treatment available. This may be warranted to some extent because the indirect utility in (2.4) can also be interpreted as reflecting uncertainty about effectiveness since it can be obtained as the value function of the expected utility maximization problem

$$\begin{aligned}
\tilde{u}_{ish}(p, v_{ish}) &= \max_{x_{ish}(\boldsymbol{\varepsilon}) \geq 0} \int u_{ish}(x_{ish}(\boldsymbol{\varepsilon}), \boldsymbol{\varepsilon}) f_{ish}(\boldsymbol{\varepsilon}) d\boldsymbol{\varepsilon} \\
& \text{s.t.} \int [px_{ish}(\boldsymbol{\varepsilon}) - p\omega_{ish}(x_{ish}(\boldsymbol{\varepsilon}), \boldsymbol{\varepsilon}) - p\gamma_{ish}(\boldsymbol{\varepsilon})] f_{ish}(\boldsymbol{\varepsilon}) d\boldsymbol{\varepsilon} = v_{ish},
\end{aligned} \tag{4.2}$$

where $f_{ish}(\boldsymbol{e})$ is the density of effectiveness for group i in state sh , and the utility and household production functions are taken to satisfy assumptions C1 and C2, respectively, for “almost every” \boldsymbol{e} , while $x_{ish}(\boldsymbol{e})$ the functional to be solved for (the stochastic optimization techniques mentioned earlier (Ermoliev et al., 2000) could be used to find an approximate solution to (4.2)). Hence, in view of the equivalence with (2.4), and for ease of exposition, we could neglect this aspect of the variability in effectiveness.

However, as formulation (4.2) supposes that the same transfer covers all \boldsymbol{e} , it cannot reflect the \mathfrak{a} -specific limits on insurability that might result from poor medication. This additional source of (supposedly idiosyncratic) uncertainty can be accommodated by including probability constraints across treatments. Health status h now refers to a possible outcome of a treatment c . The set $H_c \subseteq \{1, \dots, h\}$ denotes the possible outcomes, that may differ with respect to their insurability. Then, supposing that this treatment c has

⁸ The individual might even prefer no to repay his debt but here we assume that he will not be given the credit to have expected expenditure exceed expected revenue, given the insurability constraints, i.e. that all relevant possibilities, including that of insolvency, are anticipated by all parties.

a probability \mathbf{b}_{isch} of leading to outcome h , we can represent this additional uncertainty by including the nonnegative variables N_{is}^c and the constraints:

$$\begin{aligned} n_{ish} &= \sum_c \mathbf{b}_{isch} N_{is}^c \\ \sum_c N_{is}^c &= N_{is} \end{aligned} \tag{4.3}$$

to be inserted in (3.1) as well as (3.2).

Consumption effects on prevention (nutrition)

The impact of consumption (nutritional status and medication) on household supply was accounted for via the household production function but we did not allow for similar effects regarding prevalence. A natural way would be to include the aggregate consumption by state, and the number of individuals in the prevention function $Q_{is}(d, \sum_h X_{ish}, N_i)$, supposed to be concave and nondecreasing in the three arguments. In addition, contagion could be included by keeping the minimal prevalence convex increasing in the fraction of patients: $\tilde{P}_{is}(\sum_h n_{ish} / N_i)$. In fact, including consumption in the prevention function could be seen as the model of an individual engaged in “private” prevention. These relationships are readily introduced in the welfare program but they require an adaptation of the insurer’s program (2.8), because transfers affect consumption. Consequently, decentralization of this insurer problem to private consumption decisions (2.5) is complicated further, due to the externalities in prevention.

Treatment in successive periods: aggregate risk

Health insurance contracts are inherently dynamic, as most people have to buy insurance before the illness has revealed itself. This means that the premium should be paid ahead of the returns. A health insurer may not be willing to provide the necessary credit for this. Thos calls for a two-period model, with a non-negative premium paid in advance of insurance benefits:

$$\begin{aligned} \max_{v_{i0} \leq 0, v_{ish} \geq 0, \text{all } sh} & \tilde{u}_{i0}(p_0, v_{i0}) + \beta \sum_s P_{is} \sum_h \mathbf{p}_{ish} \tilde{u}_{ish}(p_1, v_{ish}) \\ \text{s.t.} & v_{i0} + \sum_s P_{is} \sum_h \mathbf{p}_{ish} (v_{ish} + t_i + \mathbf{f}_h) = 0 \\ & v_{ish} + \mathbf{f}_h \leq \bar{a}_{ish}, \end{aligned} \tag{4.4}$$

where β is a discount factor less than unity. In this model, the financial arrangement is less flexible than a combined savings-insurance structure, since the individual cannot borrow in period 1, and only receives nonnegative benefits. Hence, model (4.4) contains a market imperfection due to the absence of credit. However, these sign-constraints can be

interpreted as bounds on insurability (upper bound for period 0 and lower bound for period 1). Indeed, with an appropriate labeling of states and treatments with associated bounds on insurability, and definition of probabilities – and probability $1/(1+b)$ for period 0 – includes model (4.4) as a special case. Therefore, to represent the bounds on insurance, the single period formulation seems adequate.

However, there are other reasons to improve on the dynamics. We have assumed that the pool of policy holders is sufficiently large to guarantee that in the proportions defined by the probability distribution, a specified fraction of every group incurs the specified illnesses within the time period under consideration, and the insurance organization manages at least one lottery on access to treatment within the same period. Hence, for the organizer of a lottery, probabilities turn into realizations, and the aggregate outcomes of the risk pool have zero variance, enabling the insurer to guarantee that all obligations will be met. Thus, for every group i the consumer problem could be interpreted as a fully deterministic model of (true) realizations.

By contrast, if there is, say, only one ticket in a life lottery, then in the absence of insurance, patients may end up in poverty, or even die, while healthy people live in luxury. Dasgupta and Ray (1986) point to such a dualization on the labor market in developing countries, supposing that workers only get one chance in a lifetime. Srinivasan (1994) argues that this generally is unrealistic as workers usually get many chances. Keyzer (1995) specifies a model in which workers have the opportunity to avoid poverty by joining a collective insurance arrangement and abiding by it. Indeed, the insurability itself is vulnerable. If rich people are given the opportunity to bid for health services when the need arises, they might not be willing to participate in insurance arrangements. Furthermore, consumers only join mutual insurance contracts with people (or families) in the same or better health condition. Hence, they will separate from collective arrangements once they have come to know that their position is relatively favorable. Similarly, regarding the coverage of insurance, there hardly is a need to enter an insurance arrangement to deal with light forms of influenza because the consequences of the disease are not dramatic and one has many chances of incurring it: there are many lotteries or many tickets in one lottery within the period under consideration.

To represent these dynamic aspects, one would need to allow for treatment in successive periods. Improvements in longevity and quality of life are more naturally studied in a model that allows for treatment in at least two periods. For the individual, the distinction is readily accounted for by appending a date to states and labels, and by treating the probabilities in period 2 as conditional on those in period 1. For the insurer, endogenizing these probabilities amounts to solving an assignment problem with prevention measures and investments in treatment capacities determining the possibilities of providing suitable treatments. Like in the earlier prevalence function, the requirements could be made dependent on the consumption bundle enjoyed at the situation of origin, so

as to reflect the impact of consumption in the determination of conditional probabilities from one period to the next.

As soon as the number of periods exceeds one, the realized fraction becomes uncertain and defined by a probability distribution across periods i.e. to allow for aggregate risk within periods. Regarding illness, even a large collective of consumers never knows for sure that all probabilities will realize in a given period. For example, if probabilities are constant over time and independent across individuals and states (no contagion) with variance \mathbf{s}_{is}^2 in every period – the variance for a given number of tickets per periodwise lottery – then the Central Limit Theorem applies and the variance of the mean is \mathbf{s}_{is}^2/N_i . For very large N_i , the variance drops to zero and risk can be treated as idiosyncratic. For smaller values of N_i , the variance can be reduced by composition of appropriate portfolios, combining groups with different risk profiles.

To include aggregate risk in the model, we suppose that it originates from shocks in technology (transformation functions, endowment supply, and prevalence of diseases) and is denoted by the index g :

$$\begin{aligned}
& \max_{c,d,d_i,x_{gish} \geq 0, n_{gish}, \pi_{gish}, P_{gis} \geq 0, y_{gj}, \text{all } gishj} \sum_g P_g \sum_i N_i \sum_s P_{gis} \left(\sum_h \pi_{gish} \alpha_{gish} u_{gish}(x_{gish}) \right) \\
& \text{s.t.} \quad \sum_{ish} n_{gish} x_{gish} + c + d \leq \sum_{ish} n_{gish} (\omega_{gish}(x_{gish}) + \gamma_{gish}) + \sum_j y_{gj} \quad (p_g) \\
& \quad F_{gj}(y_{gj}) \leq 0 \\
& \quad \underline{P}_{gis}(d_i) N_i \leq P_{gis} N_i \\
& \quad P_{gis} N_i \leq \bar{P}_{gis} N_i \\
& \quad \sum_{is} n_{gish} \leq \bar{N}_h(c) \quad (\phi_{gh}) \quad (4.6) \\
& \quad \pi_{gish} = \frac{n_{gish}}{\sum_h n_{gish}} \\
& \quad P_{gis} = \frac{\sum_h n_{gish}}{N_i} \\
& \quad \sum_s P_{gis} = 1 \\
& \quad d_i = d \quad (\psi_i)
\end{aligned}$$

where P_g is the given probability of g , and all other probabilities are conditional on g . Prevention and investment in treatment capacity are considered part of beginning of period decisions and hence independent of g . The associated program for weight adjustment is:

$$\begin{aligned}
& \max_{n_{gish} \geq 0, v_{gish}, \text{all } gish} \sum_g P_g \sum_i \delta_i \sum_{sh} n_{gish} \tilde{u}_{gish}(p_g, v_{ish}) \\
& \text{s.t.} \quad \sum_g P_g \sum_{ish} n_{gish} (v_{gish} + t_i + \phi_{gh}) = 0 \\
& \quad n_{gish} (v_{gish} + \phi_{gh}) \leq n_{gish} \bar{a}_{gish}(p_g) \\
& \quad n_{gish} \tilde{u}_{gish}(p_g, v_{gish}) \geq n_{gish} \underline{u}_{gish} \\
& \quad \underline{P}_{gis}(d) \leq \frac{\sum_h n_{gish}}{N_i} \leq \bar{P}_{gis} \\
& \quad \sum_{sh} n_{gish} \leq N_i,
\end{aligned} \tag{4.7}$$

for given $\underline{u}_{gish} = \tilde{u}_{gish}(p_g, \max(\underline{a}_{gish}(p_g) - \mathbf{j}_{gh}, -p_g \mathbf{w}_{gish}(x_{gish}) - p_g \mathbf{g}_{gish}))$. Like in the original Arrow Debreu model with contingent markets, the prices serve to adjust supply and demand in the face of aggregate risk, and there is no need to reinsure the portfolio externally. Because of aggregate risk, insurability becomes more difficult as the shocks affect all groups simultaneously. Program (4.7) maintains perfect insurability across states g , since it keeps a single budget constraint. Limited insurability across states can be introduced by constraining the transfers. For the extreme case that no transfers are possible (a drought affecting an isolated village without any livestock or grain reserves), this can be represented by maintaining separate budgets for every g : in this case solidarity among groups is the only buffering mechanism. To represent this within the more general case, the program could be converted to one with incomplete asset markets specified via a given set of financial assets (instruments) that can be used to transport funds from $g=1$, the present, to various future states in proportions depending on the nature of each asset (Radner, 1982).

Towards a technical formulation

In the practice of health policy modeling, the effectiveness of a treatment is often measured in years of life gained from the intervention (Murray, 1999), sometimes weighted by a quality coefficient.⁹ Let $\ell_{sh}(m_{ish})$ denote the concave production function for years of life gained by treatment h in state s , using inputs m_{ish} . We also assume that the utility function is strictly concave increasing in (x, ℓ) . This extension readily fits in the definition of the indirect utility function, as in (2.4):

$$\begin{aligned}
\tilde{u}_{ish}(p, v_{ish}) &= \max_{m_{ish}, x_{ish} \geq 0} u_{is}(x_{ish}, \ell_{sh}(m_{ish})) \\
& \text{s.t.} \quad p(x_{ish} + m_{ish}) = p \mathbf{w}_{ish}(x_{ish}) + p \mathbf{g}_{ish} + v_{ish}
\end{aligned} \tag{4.8}$$

⁹ The concept offers some measure of well being in a static model but it only describes a potential gain, since the actual gain follows from later events. The labor component (commodity) of the endowment function may offer a more direct measure of health status.

and therefore agrees with the earlier specification. However, the main purpose of the concept is to guide public health oriented policies more directly. This calls for additional assumptions that permit to make the transition from general to partial equilibrium, and to postulate some generally applicable individual welfare criterion allowing planners to compute the expected utility from years gained. Yet, we note that as the number of years gained becomes endogenous in a setting with treatment in successive periods, it would not be consistent to measure utility in this unit. In practical terms, a life-saving treatment in period 1 is not worth much if the individual concerned decides to live dangerously afterwards.

The transition from general to partial equilibrium becomes possible if one is willing to suppose that one non-health related consumer good, say, x_r , enters linearly in the individual utility function and is consumed in positive amounts in the solution concerned (Willig, 1976). This means that utility can be expressed in units of this commodity, that is used as numéraire: $p_r = 1$:

$$u_{ish}(x_{ish}, \ell_{ish}) = u_{ish}(x_{ish}) + g_{sh}(\ell_{ish}) + x_{rish}, \quad (4.9)$$

where $g_{sh}(\ell)$ allows for comparability across i and is also expressed in this commodity. Under these assumptions, the optimal health plan only needs to account for health related goods and can be written:

$$\begin{aligned} & \max_{c,d,d_i,M_{ish},X_{ish} \geq 0, n_{ish}, \ell_{ish} \geq 0, \text{all } ish} \sum_{ish} (n_{ish} g_{sh}(\ell_{ish}) - pM_{ish}) - pc - pd \\ \text{s.t.} \quad & \ell_{ish} \leq L_{sh}(M_{ish}/n_{ish}) \\ & \underline{P}_{is}(d_i)N_i \leq \sum_h n_{ish} \\ & \sum_h n_{ish} \leq \bar{P}_{is}N_i \\ & \sum_{is} n_{ish} \leq \bar{N}_h(c) \\ & \sum_{sh} n_{ish} \leq N_i \\ & d_i = d \end{aligned} \quad (4.10)$$

Except for the specification of the risk aversion, this is a purely technical model that could be used to for optimal planning of the health sector. It is easy to include group specific weights. Yet the major limitation of this kind of specification is that it fails to account for individual preferences and risk aversion, and for budget restrictions: the partial equilibrium principle merely assumes that expenditure on the numéraire commodity can serve as a buffer. Hence, it cannot address the hard financial choices between expenditures on health and on other consumption.

5. Health policies

Pareto efficiency of equilibrium offers the natural reference for a discussion of regulation in the health sector, as some of the policies can be seen as instruments of implementation of such an equilibrium, while others seek to correct it. Efficiency follows directly from welfare program 4.1.

Proposition 4 (Efficiency) If in the model of proposition 1, the insurability bounds are ineffective for all consumers in all states, this equilibrium is Pareto-efficient in terms of expected utilities. If bounds are effective it is Pareto-efficient in terms of realized utilities. Furthermore, a downward shift in the prevalence $\underline{P}_{is}(d)$ and an upward shift in treatment capacity $N_h(c)$ are Pareto improving.

Proof. When bounds are ineffective, their multipliers \mathbf{m}_{sh} and \mathbf{s}_{ish} are zero. Now welfare program [P1] maximizes the weighted sum of utilities of individuals in group i , with a positive weight \mathbf{d}_i on every individual. Efficiency is immediate: the possibility of making any consumer better off without anyone being worse off contradicts optimality (see e.g. Proposition 1.8 in GK). When bounds are effective, by assumption C4(h), and since the proof of proposition shows that \mathbf{a}_{III} is positive, optimality in [P1] implies that no utility function with positive welfare weight can be made better of without anyone being worse off. By the same token, utilities with zero welfare weights are zero. Therefore, every equilibrium defines an optimum [P1] that is Pareto efficient in terms of realized utilities. Finally, to verify the effect of a shift, note that the shifts mentioned relax the constraints of welfare program (3.1), thus permitting to raise the objective while keeping the expected utility of all groups above the initial level.

Implementing the equilibrium

Even though the model has much in common with the standard competitive model, its specific features make implementation of equilibrium more difficult, and may require regulation.

First, decisions on prevention have a non-rival element because of contamination effects. There is a need to co-ordinate individual actions. Second, health insurance is more critical than any other insurance because major illnesses are either chronic or infrequent within a life time. The stakes are high, involving vast sums of money, and the patient is often incapacitated at the moment the insurance arrangement is to be invoked. Hence, health insurance is a trust good that needs to be protected against abuse: insurance companies should be reliable and lotteries for treatment should be fair. Furthermore, to maintain a risk pool of sufficient size, the relatively healthy should not be given the option of cancelling insurance contracts, and insurance companies should not be allowed

to refuse bad risks. Thus, to maintain efficiency in the expected utility (ex ante) sense, a veil of ignorance should be made to prevail, and if this is not possible, regulations should limit the flexibility of contracts. If efficiency in terms of realized utilities is all that is aimed for, this reasoning can only be justified on the basis of equity considerations. Third, the health sector is necessarily characterized by capacity constraints, physically and in terms of insurability, because patients are not infinitely risk averse, and some treatments are extremely costly, if not in absolute terms then relative to the patient's purchasing power. Like insurance, treatment is also a trust good that needs regulation, as the patient cannot determine the nature and severity of the illness, the treatment required and the adequacy of the treatment provided.

In short, interventions are needed to avoid market failures due to incomplete or asymmetric information, rather than to deviate from the competitive equilibrium. Next, we turn to more active health policies that go beyond implementation of Pareto-efficiency. We consider prohibition, subsidies and rules to guarantee equal access.

Prohibition

Some treatments might be prohibited because they are too unreliable. Prohibition could be effectuated in the model by imposing a zero bound \bar{n}_{ish} on specified treatments. The multiplier belonging to that bound would measure the marginal welfare loss and, by the same token, the minimal penalty needed to support the prohibition. If the remaining treatments, including no treatment, are insurable, the new equilibrium will be Pareto efficient in expected utilities. But this is only in appearance, as the economy with the reduced technology set could be much poorer. Clearly, program (5.1) only admits efficiency improvements via measures that offer better opportunities.

Yet, inefficiency is not the only issue to be addressed. The linearity of this program with respect to n_{ish} suggests that the restrictions imposed so far may leave room for important shifts from one group of treatments to the other. This might be a source of instability, say, with patients wandering between formal and informal medicine, and penalties on uninsurable treatments might, despite the welfare loss they cause, serve to reduce this instability.

This also illustrates that focused technical progress which improves the quality of life of the patient, expressed in the model either via the utility of via the household production function, and institutional progress that relaxes the bounds on insurability will raise the surplus of the target groups (see (3.8)). In this connection, programs (3.1) and (3.2) illustrate that it may not be possible to offer treatment to all patients for all diseases. Formally, when some groups are unable to pay for treatment, imposition of the restriction $\bar{n}_{isI} = 0$ causes infeasibility in either (3.1) or (3.2) or both, and non-existence of equilibrium. This might be resolved through solidarity transfers among groups, and, if the economy under consideration is too poor, through foreign aid represented as a shift in the

technology. But if no such aid is available, the difficult question is whether it is still acceptable to help very few people than no one at all. Helping creates some hope for all, but disappointment for many. This is the HIV-dilemma referred to earlier.

Subsidies

Besides relaxing individual budget constraints, solidarity transfers between groups may also serve to link their respective risk pools. Suppose that transfers are expressed via a given schedule of insurance benefits \mathbf{b}_{ish} , financed from taxes T_i :

$$\begin{aligned}
 & \max_{n_{ish} \geq 0, v_{ish}, \text{all } sh} \sum_{sh} n_{ish} \tilde{u}_{ish}(p, v_{ish} + \beta_{ish}) \\
 \text{s.t.} \quad & \sum_{sh} n_{ish} (v_{ish} + t_i + \phi_h) + T_i = 0 & (\lambda_i) \\
 & n_{ish} (v_{ish} + \phi_h) \leq n_{ish} \bar{a}_{ish}(p) & (\mu_{ish}) \\
 & n_{ish} \tilde{u}_{ish}(p, v_{ish}) \geq n_{ish} \underline{u}_{ish} & (\sigma_{ish}) \\
 & \sum_h n_{ish} \\
 & \underline{P}_i(d) \leq \frac{\sum_h n_{ish}}{N_i} \leq \bar{P}_i \\
 & \sum_{sh} n_{ish} = N_i.
 \end{aligned} \tag{5.1}$$

A possible policy question is how to keep benefits sufficiently low to ensure that no healthy individuals register as sick. Since a marginal increase in subsidy \mathbf{b}_{ish} raises the marginal utility obtained from a transfer to that sh , it does not lead to a drop in the number of people moving to that state. Thus, subsidies on benefits affect allocations and may create disability traps, whereby individuals are reluctant to join the working force, because the sickness benefits are so generous. In a similar vein, if social transfers are made to a state with effective lower bounds on insurability, i.e. from which the patient cannot transfer income to any other state, they become contingent on reaching that state. Hence, they have same effect as a subsidy. For example, patients may seek treatment in a luxurious sanatorium, even though they would prefer to receive in cash the money spent on their treatment. Conversely, individuals may decide to leave the relative safety of their birthplace, with adequate social security, and migrate to locations where they earn more but lack social security. This amounts to opting for limited insurability. It cannot be modeled by supposing limited risk aversion, since even an almost risk neutral individual will perfectly smoothen marginal utility whenever possible.

Equal access restrictions

The tendency towards specialization in (5.1) indicates that, income groups with a higher welfare weight (the rich) will enjoy priority. Transfers would have to be very large to compensate for this. In practice, equal access is often assured through direct regulations,

that simply require from doctors and other medical staff not to discriminate between patients, even though the accommodation of the patient's room in the hospital may differ. This requirement can be represented in programs (3.1) and (3.2) via constraints

$$n_{ish} = \mathbf{x}_{sh} \mathbf{q}_{ish} N_i, \quad (\mathbf{t}_{ish}) \quad (5.2)$$

for specified sh combinations, excluding non-treatment ($h=1$), where \mathbf{x}_{sh} is an endogenous variable that maintains equality of chances and \mathbf{q}_{ish} a fixed parameter. This parameter might also be used to represent the effect of awarding privileged treatment to a particular group i.e. to enforce inequity. Alternatively, one might impose equal probabilities independently of the illness.

The shadow price associated to equal access restrictions will generally be higher for rich consumers because their marginal utility of expenditures is lower. This creates the channel through which the rich are induced to support public facilities, since this is the only way for them to receive better treatment. The resulting individual contribution will be $f_h + t_i + \mathbf{t}_{ish}$. Likewise, it is possible to insert further equity restrictions in this contribution, if only so as to represent the administrative difficulties of differentiating among groups i . This would amount to an indirect tax on n_{ish} , and act as a distortion in allocating individuals, even though for fixed n_{ish} it would be a lump sum payment.

6. Conclusion

The health sector is characterized by uncertainty and scarcity. By allowing for endogenous adjustment in the probabilities faced by individuals and by including bounds on insurability, the general equilibrium model specified in this paper provides a framework to represent many of the dilemma's of health policies. It permits to identify the interventions for efficient regulation and also provides a tool to analyze the effect of subsidies, prohibitions, and restriction that keep equal access to treatment. It also suggests that efficiency improvements are more naturally effectuated by technological advances which reduce the prevalence of illnesses and improve the effectiveness of a treatment than by prohibition of inferior or uninsurable treatments.

The model offers a computational framework to quantify various effects. Because of its relatively general assumptions on preferences and technology a numerical application could make use of the existing empirical literature on prevalence of illness, and the cost and effectiveness of prevention and treatment. Although the general equilibrium model covers all sectors of the economy, such an application would not necessarily need an elaborate representations of the rest of the economy: prices of non-health related goods can be taken as given and the associated commodity balances can be relaxed by allowing for net inflows. A possible complication is dimensionality, as the

number of consumer groups, firms, commodities, states and medical treatments, and possibly time periods combine into a problem of very large scale. This needs further investigation.

At the applied level, the health sector modeler still has to choose between accepting the simplifications of partial analysis that permit to sidestep the problem of pricing access to treatment and gains from prevention on the one hand, and accounting for individual budget constraints as well as differences in preferences and risk aversion on the other. Moreover, the modeler needs ethical guidance. When an agency only has the financial means to help part of the patients, there is no clear moral principle establishing that it should opt for Pareto-efficiency, i.e. help whomever it can. In the tantalizing choice of a poor developing country with HIV between extending the life a selected groups and not treating anyone, the calculus of fair lotteries falls silent.

References

- Dasgupta, P., and D. Ray (1986) 'Inequality as determinant of malnutrition and unemployment: Theory', *Economic Journal*, 96: 1011-34.
- Debreu, G. (1959) *Theory of Value*. New York: Wiley.
- Ermoliev, Yu., M.A. Keyzer, and V. Norkin (2000) 'Global convergence of the stochastic tatonnement process,' *Journal of Mathematical Economics*, 34: 173-190.
- Ginsburgh, V., and M.A. Keyzer (1997) *The structure of applied general equilibrium models*. Cambridge MA: MIT Press.
- Hurley, J. (2000) 'An overview of the normative economics of the health sector,' Chapter 2 in *Handbook of Health Economics*, Vol. 1A, A.J. Culyer and J.P. Newhouse, eds. Amsterdam: North Holland.
- Jack, W. (1999) *Principles of health economics for developing countries*. Washington DC: World Bank Institute.
- Keyzer, M.A. (1995) 'Social security in a general equilibrium model with migration', in N.S.S. Narayana and A. Sen, eds., *Poverty, environment and economic development*. Bangalore: Interline Publishers.
- Murray, Ch.J.L., and A.D. Lopez, eds. (1999) *The global burden of disease*. Cambridge, MA: Harvard School of Public Health.
- Negishi T. (1960) "Welfare economics and existence of equilibrium for a competitive economy," *Metroeconomica* 12:92-97.
- Radner, R., (1982) *Equilibrium under uncertainty*. In K.J. Arrow and M.D. Intrilligator, eds., *Handbook of Mathematical Economics*, Vol. 2. Amsterdam: North Holland.
- Srinivasan, T.N. (1994) 'Destitution: A discourse', *Journal of Economic Literature*, 32: 1842-1855.
- Willig, R. (1976) 'Consumer's surplus without apology,' *American Economic Review*, 66: 589-597.
- Zweifel, P., and F. Breyer (1997) *Health Economics*. Oxford: Oxford University Press.

The Centre for World Food Studies (Dutch acronym SOW-VU) is a research institute related to the Department of Economics and Econometrics of the Vrije Universiteit Amsterdam. It was established in 1977 and engages in quantitative analyses to support national and international policy formulation in the areas of food, agriculture and development cooperation.

SOW-VU's research is directed towards the theoretical and empirical assessment of the mechanisms which determine food production, food consumption and nutritional status. Its main activities concern the design and application of regional and national models which put special emphasis on the food and agricultural sector. An analysis of the behaviour and options of socio-economic groups, including their response to price and investment policies and to externally induced changes, can contribute to the evaluation of alternative development strategies.

SOW-VU emphasizes the need to collaborate with local researchers and policy makers and to increase their planning capacity.

SOW-VU's research record consists of a series of staff working papers (for mainly internal use), research memoranda (refereed) and research reports (refereed, prepared through team work).

Centre for World Food Studies
SOW-VU
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

Telephone (31) 20 - 44 49321
Telefax (31) 20 - 44 49325
Email pm@sow.econ.vu.nl
www <http://www.sow.econ.vu.nl/>